ODD

Answers to Odd-Numbered Problems, 4th Edition of Games and Information, Rasmusen

CHAPTER 8: Further Topics in Moral Hazard

26 March 2005. 12 November 2005. Erasmuse@indiana.edu. Http://www.rasmusen.org.

This appendix contains answers to the odd-numbered problems in the fourth edition of *Games and Information* by Eric Rasmusen, which I am working on now and perhaps will come out in 2006. The answers to the even- numbered problems are available to instructors or self-studiers on request to me at Erasmuse@indiana.edu.

Other books which contain exercises with answers include Bierman & Fernandez (1993), Binmore (1992), Fudenberg & Tirole (1991a), J. Hirshleifer & Riley (1992), Moulin (1986), and Gintis (2000). I must ask pardon of any authors from whom I have borrowed without attribution in the problems below; these are the descendants of problems that I wrote for teaching without careful attention to my sources.

PROBLEMS FOR CHAPTER 8: Further Topics in Moral Hazard

8.1. Monitoring with error (easy)
An agent has a utility function $U = \sqrt{w} - \alpha e$, where $\alpha = 1$ and e is either 0 or 5. His reservation utility level is $\overline{U} = 9$, and his output is 100 with low effort and 250 with high effort. Principals are risk neutral and scarce, and agents compete to work for them. The principal cannot condition the wage on effort or output, but he can, if he wishes, spend five minutes of his time, worth 10 dollars, to drop in and watch the agent. If he does that, he observes the agent *Daydreaming* or *Working*, with probabilities that differ depending on the agent's effort. He can condition the wage on those two things, so the contract will be $\{\underline{w}, \overline{w}\}$. The probabilities are given by Table 1.

Table 1: Monitoring with Error

	Probability of	
Effort	*Daydreaming*	*Working*
$Low(e = 0)$	0.6	0.4
$High(e = 5)$	0.1	0.9

(a) What are profits in the absence of monitoring, if the agent is paid enough to make him willing to work for the principal? Answer. Without monitoring, effort is low. The participation constraint is $\sqrt{w} - 0 \geq 9$, so $w = 81$. Output is 100, so profit is 19.

(b) Show that high effort is efficient under full information.

Answer. High effort yields output of 250. $\overline{U} \geq \sqrt{w} - \alpha e$ or $9 = \sqrt{w} - 5$ is the participation constraint, so $14 = \sqrt{w}$ and $w = 196$. Profit is then 54. This is superior to the profit of 19 from low effort (and the agent is no worse off), so high effort is more efficient.

(c) If $\alpha = 1.2$, is high effort still efficient under full information?

Answer. If $\alpha = 1.2$, then the wage must rise to 225, for profits of 25, so high effort is still efficient. The wage must rise to 225 because the participation constraint becomes $\sqrt{w} - 1.2(5) \geq 9$.

(d) Under asymmetric information, with $\alpha = 1$, what are the participation and incentive compatibility constraints?

Answer. The incentive compatibility constraint is

$$0.6\sqrt{\underline{w}} + 0.4\sqrt{\overline{w}} \leq 0.1\sqrt{\underline{w}} + 0.9\sqrt{\overline{w}} - 5.$$

The participation constraint is $9 \leq 0.1\sqrt{\underline{w}} + 0.9\sqrt{\overline{w}} - 5$.

(e) Under asymmetric information, with $\alpha = 1$, what is the optimal contract?

Answer. From the participation constraint, $14 = 0.1\sqrt{\underline{w}} + 0.9\sqrt{\overline{w}}$, and $\sqrt{\overline{w}} = \frac{14}{0.9} - (\frac{1}{9})\sqrt{\underline{w}}$. The incentive compatibility constraint tells us that $0.5\sqrt{\overline{w}} = 5 + 0.5\sqrt{\underline{w}}$, so $\sqrt{\overline{w}} = 10 + \sqrt{\underline{w}}$. Thus,

$$10 + \sqrt{\underline{w}} = 15.6 - 0.11\sqrt{\underline{w}} \quad (1)$$

and $\sqrt{\underline{w}} = 5.6/1.11 = 5.05$. Thus, $\underline{w} = 25.5$. It follows that $\sqrt{\overline{w}} = 10 + 5.05$, so $\overline{w} = 226.5$.

8.3. Bankruptcy Constraints

A risk- neutral principal hires an agent with utility function $U = w - e$ and reservation utility $\overline{U} = 5$. Effort is either 0 or 10. There is a bankruptcy constraint: $w \geq 0$. Output is given by Table 4.

Table 4: Bankruptcy

	Probability of Output of		
Effort	0	400	Total
Low ($e = 0$)	0.5	0.5	1
High ($e = 10$)	0.2	0.8	1

(a) What would be the agent's effort choice and utility if he owned the firm?

Answer. $e = 10$, because expected output is then 360 instead of the 200 with low effort, and the agent's utility is 350 instead of 200.

(b) If agents are scarce and principals compete for them what will be the agent's contract under full information? His utility?

Answer. Effort is high, as found in part (a). The wage is 360 for high effort and 0 for low (though there are other possibilities). Agent utility is 350.

(c) If principals are scarce and agents compete to work for them, what will the contract be under full information? What will the agent's utility be?

Answer. Because principals are scarce, $U = \overline{U} = 5$. Effort is high. The wage is 15 if effort is high, and 0 if it is low.

(d) If principals are scarce and agents compete to work for them, what will the contract be when the principal cannot observe effort? What will the payoffs be for each player?

Answer. An efficiency wage must be paid so that the incentive compatibility constraint of part (d) is satisfied. The participation constraint is thus not binding. The low wage will be 0, since the principal wants to make the gap as big as possible between the low wage and the high wage. The high wage must equal 25 to get incentive compatibility. Hence,

$$U = 0.1(0) + 0.9(25) - 10 = 12.5 \tag{2}$$

$\pi(H) = 337.5(= 0.1(0-0) + 0.9(400-25))$. This exceeds $\pi(L) = 195(= 0.5(0-5) + 0.5(400-5))$.

(e) Suppose there is no bankruptcy constraint. If principals are the scarce factor and agents compete to work for them, what will the contract be when the principal cannot observe effort? What will the payoffs be for principal and agent?

Answer. Since agents are risk neutral, selling the store works well. The expected wage must be 15 for the agent so that $U = \overline{U} = 5$, and an incentive compatibility constraint must be satisfied to obtain high effort:

$$0.5w(0) + 0.5w(400) \leq 0.1w(0) + 0.9w(400) - 10, \tag{3}$$

which can be rewritten as $w(400) - w(0) \geq 25$. Many contracts can ensure this. One is to sell the store for 360 minus 10 for the high effort minus 5 for the opportunity cost, which is equivalent to letting the agent keep all the output for a lump-sum payment of 345: $w(0) = 0 + 15 - 360 = -345$ and $w(400) = 400 + 15 - 360 = 55$, which averages to an expected wage of 15 and an expected utility of 5. The principal's payoff is 345.

8.5. Efficiency Wages and Risk Aversion (see Rasmusen [1992c])
In each of two periods of work, a worker decides whether to steal amount v, and is detected with probability α and suffers legal penalty p if he, in fact, did steal. A worker who is caught stealing can also be fired, after which he earns the reservation wage w_0. If the worker does not steal, his utility in the period is $U(w)$; if he steals, it is $U(w+v) - \alpha p$, where $U(w_0 + v) - \alpha p > U(w_0)$. The worker's marginal utility of income is diminishing: $U' > 0$, $U'' < 0$, and $lim_{x\to\infty} U'(x) = 0$. There is no discounting. The firm definitely wants to deter stealing in each period, if at all possible.

(a) Show that the firm can indeed deter theft, even in the second period, and, in fact, do so with a second-period wage w_2^* that is higher than the reservation wage w_0.

Answer. It is easiest to deter theft in the first period, since a high second-period wage increases the penalty of being fired. If w_2 is increased enough, however, the marginal utility of income becomes so low that $U(w_2 + v)$ and $U(w_2)$ become almost identical, and the difference is less than αP, so theft is deterred even in the second period.

(b) Show that the equilibrium second-period wage w_2^* is higher than the first-period wage w_1^*.

Answer. We already determined that $w_2 > w_0$. Hence, the worker looks hopefully towards being employed in period 2, and in Period 1 he is reluctant to risk his job by stealing. This means that he can be paid less in Period 1, even though he may still have to be paid more than the reservation wage.

8.7. Machinery

Mr. Smith is thinking of buying a custom- designed machine from either Mr. Jones or Mr. Brown. This machine costs 5000 dollars to build, and it is useless to anyone but Smith. It is common knowledge that with 90 percent probability the machine will be worth 10,000 dollars to Smith at the time of delivery, one year from today, and with 10 percent probability it will only be worth 2,000 dollars. Smith owns assets of 1,000 dollars. At the time of contracting, Jones and Brown believe there is there is a 20 percent chance that Smith is actually acting as an "undisclosed agent" for Anderson, who has assets of 50,000 dollars.

Find the price be under the following two legal regimes: (a) An undisclosed principal is not responsible for the debts of his agent; and (b) even an undisclosed principal is responsible for the debts of his agent. Also, explain (as part [c]) which rule a moral hazard model like this would tend to support.

Answer. (a) The zero profit condition, arising from competition between Jones and Brown, is

$$-5000 + 0.9P + 0.1(1000) = 0, \tag{4}$$

because Smith will only pay for the machine with probability 0.9, and otherwise will default and only pay up to his wealth, which is 1. This yields $P \approx 5,444$.

(b) If Anderson is responsible for Smith's debts, then Smith will pay the 5,000 dollars. Hence, zero profits require

$$-5000 + 0.9P + 0.1(0.2)P + 0.1(0.8)(1000) = 0, \tag{5}$$

which yields $P \approx 5,348$.

(c) Moral hazard tends to support rule (b). This is because it reduces bankruptcy and the agent will be more reluctant to order the machine when there is a high chance it is unprofitable. In the model as constructed, this does not arise, because there is only one type of agent, but more generally it would, because there would be a continuum of types of agents, and some who would buy the machine under rule (b) would find it too expensive under rule (a).

Even in the model as it stands, rule (a) leads to the inefficient outcome that a machine worth 2,000 to Smith is not give to Smith. Rather, he pays his wealth and lets the seller keep the machine, which is inefficient since the machine really is worth 2000 to Smith.

This is a question about zero-profit prices. Guessing would have been a good idea here: it is very intuitive that the price would always be above \$5,000, and that it would be higher if the principal never had to cover the agent's debts. You should be able to tell that $P > 10,000$ is impossible, because Smith would never pay it. Also, the sellers compete, so it is their profits that provide a participation constraint, not the benefit to the buyer.