ODD

Answers to Odd-Numbered Problems, 4th Edition of Games and Information, Rasmusen

PROBLEMS FOR CHAPTER 12: Bargaining

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This appendix contains answers to the odd-numbered problems in the fourth edition of *Games and Information* by Eric Rasmusen, which I am working on now and perhaps will come out in 2006. The answers to the even- numbered problems are available to instructors or self- studiers on request to me at Erasmuse@indiana.edu.

Other books which contain exercises with answers include Bierman & Fernandez (1993), Binmore (1992), Fudenberg & Tirole (1991a), J. Hirshleifer & Riley (1992), Moulin (1986), and Gintis (2000). I must ask pardon of any authors from whom I have borrowed without attribution in the problems below; these are the descendants of problems that I wrote for teaching without careful attention to my sources.

12.1. A Fixed Cost of Bargaining and Grudges
Smith and Jones are trying to split 100 dollars. In bargaining round 1, Smith makes an offer at cost 0, proposing to keep S_1 for himself and Jones either accepts (ending the game) or rejects. In round 2, Jones makes an offer at cost 10 of S_2 for Smith and Smith either accepts or rejects. In round 3, Smith makes an offer of S_3 at cost c, and Jones either accepts or rejects. If no offer is ever accepted, the 100 dollars goes to a third player, Dobbs.

(a) If $c = 0$, what is the equilibrium outcome?

Answer. $S_1 = 100$ and Jones accepts it. If Jones refused, he would have to pay 10 to make a proposal that Smith would reject, and then Smith would propose $S_3 = 100$ again. $S_1 < 100$ would not be an equilibrium, because Smith could deviate to $S_1 = 100$ and Jones would still be willing to accept.

(b) If $c = 80$, what is the equilibrium outcome?

Answer. If the game goes to Round 3, Smith will propose $S_3 = 100$ and Jones will accept, but this will cost Smith 80. Hence, if Jones proposes $S_2 = 20$, Smith will accept it, leaving 80 for Jones—who would, however pay 10 to make his offer. Hence, in Round 1 Smith must offer $S_1 = 30$ to induce Jones to accept, and that will be the equilibrium outcome.

(c) If $c = 10$, what is the equilibrium outcome?

Answer. If the game goes to Round 3, Smith will propose $S_3 = 100$ and Jones will accept, but this will cost Smith 10. Hence, if Jones proposes $S_2 = 90$, Smith will accept it, leaving 10 for Jones—who would, however pay 10 to make his offer. Hence, in Round 1 Smith need only offer $S_1 = 100$ to induce Jones to accept, and that will be the equilibrium outcome.

(d) What happens if $c = 0$, but Jones is very emotional and would spit in Smith's face and throw the 100 dollars to Dobbs if Smith proposes $S = 100$? Assume that Smith knows Jones's personality perfectly.

Answer. However emotional Jones may be, there is some minimum offer M that he would accept, which probably is less than 50 (but you never know—some people think they are entitled to everything, and one could imagine a utility function such that Jones would refuse $S = 5$ and prefer to bear the cost 10 in the second round in order to get the whole 100 dollars). The equilibrium will be for Smith to propose exactly S-M in Round 1, and for Jones to accept.

12.3. The Nash Bargaining Solution
Smith and Jones, shipwrecked on a desert island, are trying to split 100 pounds of cornmeal

and 100 pints of molasses, their only supplies. Smith's utility function is $U_s = C + 0.5M$ and Jones' is $U_j = 3.5C + 3.5M$. If they cannot agree, they fight to the death, with $U = 0$ for the loser. Jones wins with probability 0.8.

(a) What is the threat point?

<u>Answer.</u> The threat point gives the expected utility for Smith and Jones if they fight. This is 560 for Jones (= 0.8(350 + 350) + 0), and 30 for Smith (=0.2(100+50) + 0).

(b) With a 50-50 split of the supplies, what are the utilities if the two players do not recontract? Is this efficient?

underline*Answer.* The split would give the utilities $U_s = 75$ (= 50 + 25) and $U_j = 350$. If Smith then traded 10 pints of molasses to Jones for 8 pounds of cornmeal, the utilities would become $U_s = 78$ (= 58+20) and $U_j = 357$ (=3.5(60) + 3.5(42)), so both would have gained. The 50-50 split is not efficient.

(c) Draw the threat point and the Pareto frontier in utility space (put U_s on the horizontal axis).

<u>Answer.</u> See Figure A12.1.

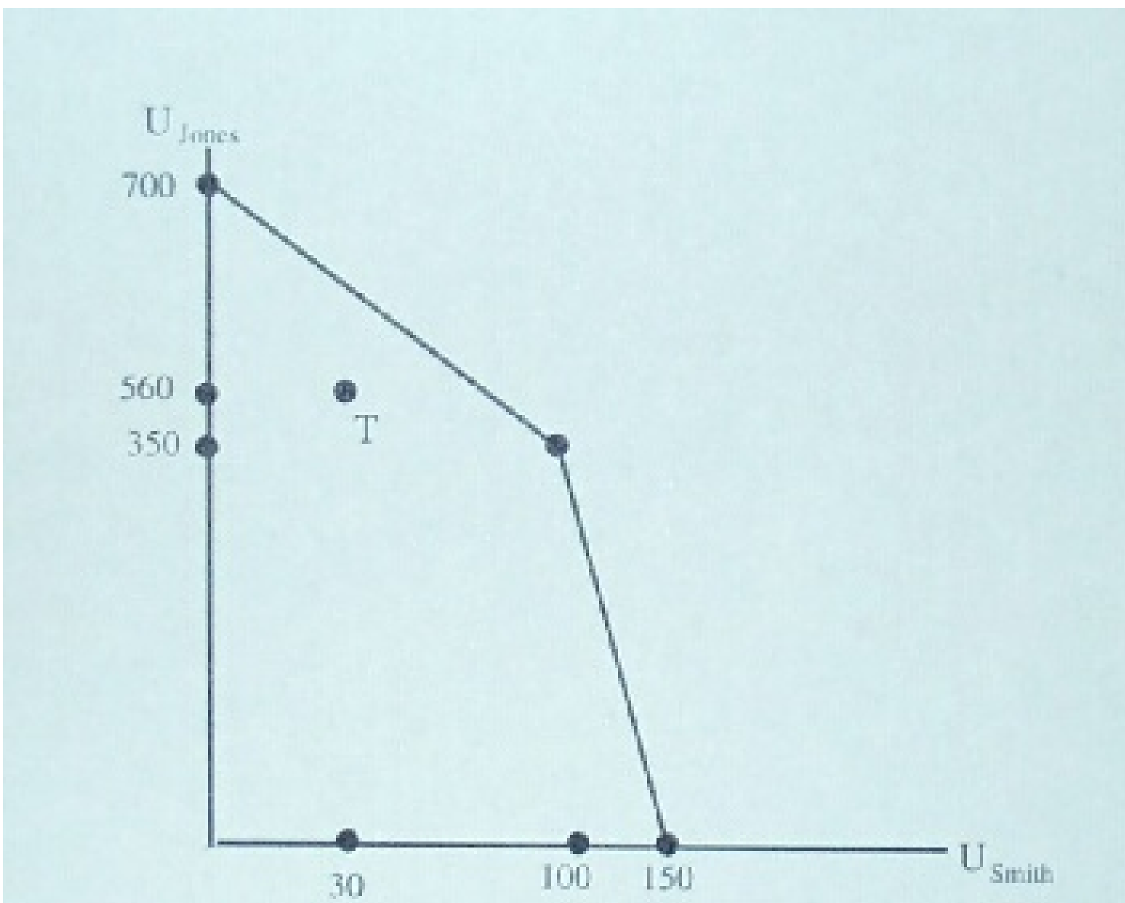

Figure A12.1: The Threat Point and Pareto Frontier

To draw the diagram, first consider the extreme points. If Smith gets everything, his utility is 150 and Jones's is 0. If Jones gets everything, his utility is 700 and Smith's is 0. If we start at (150,0) and wish to efficiently help Jones at the expense of Smith, this is done by giving Jones some molasses, since Jones puts a higher relative value on molasses. This can be done until Jones has all the molasses, at utility point (100, 350). Beyond there, one must take cornmeal away from Smith if one is to help Jones further, so the Pareto frontier acquires a flatter slope.

(d) According to the Nash bargaining solution, what are the utilities? How are the goods split?

<u>*Answer.*</u> To find the Nash bargaining solution, maximize $(U_s - 30)(U_j - 560)$. Note from the diagram that it seems the solution will be on the upper part of the Pareto frontier, above (100,350), where Jones is consuming all the molasses, and where if Smith loses one utility unit, Jones gets 3.5. If we let X denote the amount of cornmeal that Jones gets, we can rewrite the problem as

$$\overset{Maximize}{X} \quad (100 - X - 30)(350 + 3.5X - 560) \tag{1}$$

This maximand equals $(70 - X)(3.5X - 210) = -14,700 + 455X - 3.5X^2$. The first order condition is $455 - 7X = 0$, so $X^* = 65$. Thus, Smith gets 35 pounds of cornmeal, Jones gets 65 pounds of cornmeal and 100 of molasses, and $U_s = 35$ and $U_j = 577.5$.

(e) Suppose Smith discovers a cookbook full of recipes for a variety of molasses candies and corn muffins, and his utility function becomes $U_s = 10C + 5M$. Show that the split of goods in part (d) remains the same despite his improved utility function.

<u>*Answer.*</u> The utility point at which Jones has all the molasses and Smith has the molasses is now (1000, 350), since Smith's utility is (10) (100). Smith's new threat point utility is $300 (= 0.2((10)(100) + (5)(100))$. Thus, the Nash problem of equation (1) becomes

$$\overset{Maximize}{X} \quad (1000 - 10X - 300)(350 + 3.5X - 560). \tag{2}$$

But this maximand is the same as $(10)(100 - X - 30)(350 + 3.5X - 560)$, so it must have the same solution as was found in part (d) .

12.5. A Fixed Cost of Bargaining and Incomplete Information

Smith and Jones are trying to split 100 dollars. In bargaining round 1, Smith makes an offer at cost c, proposing to keep S_1 for himself. Jones either accepts (ending the game) or rejects. In round 2, Jones makes an offer of S_2 for Smith, at cost 10, and Smith either accepts or rejects. In round 3, Smith makes an offer of S_3 at cost c, and Jones either accepts or rejects. If no offer is ever accepted, the 100 dollars goes to a third player, Parker.

(a) If $c = 0$, what is the equilibrium outcome?

<u>*Answer.*</u> $S_1 = 100$ and Jones accepts it. If Jones refused, he would have to pay 10 to make a proposal that Smith would reject, and then Smith would propose $S_3 = 100$ again. $S_1 < 100$ would not be an equilibrium, because Smith could deviate to $S_1 = 100$ and Jones would still be willing to accept .

(b) If $c = 80$, what is the equilibrium outcome?

<u>*Answer.*</u> If the game goes to Round 3, Smith will propose $S_3 = 100$ and Jones will accept, but this will cost Smith 80. Hence, if Jones proposes $S_2 = 20$, Smith will accept it, leaving 80 for Jones—who would, however, pay 10 to make his offer. Hence, in Round 1 Smith must offer $S_1 = 30$ to induce Jones to accept, which will be the equilibrium outcome.

(c) If Jones' priors are that $c = 0$ and $c = 80$ are equally likely, but only Smith knows the true value, what are the players' equilibrium strategies in rounds 2 and 3? (that is: what are S_2 and S_3, and what acceptance rules will each player use?)

<u>*Answer.*</u> Jones proposes $S_2 = 20$ and accepts $S_3 \leq 100$. Smith accepts $S_2 \geq 20$ if $c = 80$ and $S_2 \geq 100$ if $c = 0$, and proposes $S_3 = 100$ regardless of c.

The rationale behind the equilibrium strategies is as follows. In Round 3, either type of Smith does best by proposing a share of 100, and Jones might as well accept. In Round 2, anything but $S_2 = 100$ would be rejected by Smith if $c = 0$, so Jones should give up on that and offer $S_2 = 20$, which would be accepted if $c = 80$ because if that type of Smith were to wait, he would have to pay 80 to propose $S_3 = 100$.

(d) If Jones' priors are that $c = 0$ and $c = 80$ are equally likely, but only Smith knows the true value, what are the equilibrium strategies for round 1? (Hint: the equilibrium uses mixed strategies.)

<u>*Answer.*</u> Smith's equilibrium strategy is to offer $S_1 = 100$ with probability 1 if $c = 0$ and probability $\frac{1}{7}$ if $c = 80$; to offer $S_1 = 30$ with probability 6/7 if $c = 80$. Jones accepts $S_1 = 100$ with probability $\frac{1}{8}$, rejects $S_1 \in (30, 100)$, and accepts $S_1 \leq 30$. Out of equilibrium, a supporting belief is for Jones to believe that if S_1 equals neither 30 nor 100, then $Prob(c = 80) = 1$.

In Round 1, if $c = 0$, Smith should propose $S_1 = 100$, since he can wait until Round 3 and get that anyway at zero extra cost. There is no pure strategy equilibrium, because if $c = 80$, Smith would pretend that $c = 0$ and propose $S_1 = 100$ if Jones would accept that. But if Jones accepts only with probability θ, then Smith runs the risk of only getting 20 in the second period, less than $S_1 = 30$, which would be accepted by Jones with probability 1. Similarly, if Smith proposes $S_1 = 100$ with probability γ when $c = 80$, Jones can either accept it, or wait, in which case Jones might either pay a cost of 10 and end up with $S_3 = 100$ anyway, or get Smith to accept $S_2 = 20$.

The probability γ must equate Jones's two pure-strategy payoffs. Using Bayes's Rule for the probabilities in (4), the payoffs are

$$\pi_j(accept\ S_1 = 100) = 0 \tag{3}$$

and

$$\pi_j(reject\ S_1 = 100) = -10 + \left(\frac{0.5\gamma}{0.5\gamma + 0.5}\right)(80) + \left(\frac{0.5}{0.5\gamma + 0.5}\right)(0), \tag{4}$$

which yields $\gamma = \frac{1}{7}$.

The probability θ must equate Smith's two pure-strategy payoffs:

$$\pi_s(S_1 = 30) = 30 \tag{5}$$

and

$$\pi_s(S_1 = 100) = \theta 100 + (1 - \theta)20, \tag{6}$$

which yields $\theta = \frac{1}{8}$.

12.7. Myerson-Satterthwaite

The owner of a tract of land values his land at v_s and a potential buyer values it at v_b. The buyer and seller do not know each other's valuations, but guess that they are uniformly distributed between 0 and 1. The seller and buyer suggest p_s and p_b simultaneously, and they have agreed that the land will be sold to the buyer at price $p = \frac{(p_b + p_s)}{2}$ if $p_s \leq p_b$.

The actual valuations are $v_s = 0.2$ and $v_b = 0.8$. What is one equilibrium outcome given these valuations and this bargaining procedure? Explain why this can happen.

Answer. This game is Bilateral Trading III. It has multiple equilibria, even for this one pricing mechanism.

The One Price Equilibrium described in Chapter 12 is one possibility. The Buyer offers $p_b = x$ and the Seller offers $p_s = x$, with $x \in [.2, .8]$, so that $p = x$. If either player tries to improve the price from his point of view, he will lose all gains from trade. And he of course will not want to give the other player a better price when that does not increase the probability of trade.

A degenerate equilibrium is for the Buyer to offer $p_b = 0$ and the Seller to offer $p_s = 1$, in which case trade will not occur. Neither player can gain by unilaterally altering his strategy, which is why this is a Nash equilibrium. You will be able to think of other degenerate no- trade equilibria too.

The Linear Equilibrium described in Chapter 12 uses the following strategies:

$$p_b = \frac{2}{3}v_b + \frac{1}{12}$$

and

$$p_s = \frac{2}{3}v_s + \frac{1}{4}.$$

Substituting in our v_b and v_s yields a buyer price of $p_b = (2/3)(.8) + 1/12 = 192/360 + 30/360 = 222/360$ and a seller price of $p_s = (2/3)(.2) + 1/4 = 16/120 + 30/120 = 23/60 = 138/360$. Trade will occur, and at a price halfway between these values, which is $p = (1/2)(222 + 138)/360 = 1/2$.

This will be an equilibrium because although we have specified $v_s$and v_b , the players do not both know those values till after the mechanism is played out.