ODD

Answers to Odd-Numbered Problems, 4th Edition of Games and Information, Rasmusen

CHAPTER 9 Adverse Selection

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This appendix contains answers to the odd-numbered problems in the fourth edition of *Games and Information* by Eric Rasmusen, which I am working on now and perhaps will come out in 2005. The answers to the even- numbered problems are available to instructors or self-studiers on request to me at Erasmuse@indiana.edu.

Other books which contain exercises with answers include Bierman & Fernandez (1993), Binmore (1992), Fudenberg & Tirole (1991a), J. Hirshleifer & Riley (1992), Moulin (1986), and Gintis (2000). I must ask pardon of any authors from whom I have borrowed without attribution in the problems below; these are the descendants of problems that I wrote for teaching without careful attention to my sources.

9.1. Insurance with Equations and Diagrams

The text analyzes Insurance Game III using diagrams. Here, let us use equations too. Let $U(t) = log(t)$.

(a) Give the numeric values (x, y) for the full-information separating contracts C_3 and C_4 from Figure 9.6. What are the coordinates for C_3 and C_4?

Answer. C_3 yields zero profits, so $0.25x + 0.75(x - y) = 0$, and it insures fully, so $12 - x = y - x$. Put together, these give $y = 4x/3$ and $y = 12$, so $x^* = 9$ and $y^* = 12$.

$C_3 = (3, 3)$ because 12-9 = 3.

C_4 yields zero profits, so $0.5x + 0.5(x - y) = 0$, and it fully insures, so $12 - x = y - x$. Put together, these give $y = 2x$ and $y = 12$, so $x^* = 6$ and $y^* = 12$.

$C_4 = (6, 6)$ because 12-6 = 6.

(b) Why is it not necessary to use the $U(t) = log(t)$ function to find the values?

Answer. We know there is full insurance at the first-best with any risk-averse utility function, so the precise function does not matter.

(c) At the separating contract under incomplete information, C_5, $x = 2.01$. What is y? Justify the value 2.01 for x. What are the coordinates of C_5?

Answer. At C_5, the incentive compatibility constraints require that $0.5x + 0.5(x - y) = 0$, so $y = 2x$; and $\pi_u(C_5) = \pi_u(C_3)$, so $0.25log(12 - x) + 0.75log(y - x) = 0.25log(3) + 0.75log(3)$. Solving these equations yields $x^* = 2.01$ and $y = 4.02$.

$C_5 = (9.99, 2.01)$ because 9.99=12-2.01 and $2.01 = 4.02 - 2.01$.

(d) What is a contract C_6 that might be profitable and that would lure both types away from C_3 and C_5?

Answer. One possibility is $C_6 = (8, 3)$, or $(x = 4, y = 7)$. The utility of this to the Highs is 1.59 $(= 0.5log(8) + 0.5log(3))$, compared to 1.57 $(=0.5log(10.99) + 0.5log(2.01))$, so the High's prefer it to C_5, and that means the Lows will certainly prefer it. If there are not many Lows, the contract can make a profit, because if it is only Highs, the profit is 0.5 $(=0.5(4) + 0.5(4 - 7))$.

9.3. Finding the Mixed-Strategy Equilibrium in a Testing Game

Half of high school graduates are talented, producing output $a = x$, and half are untalented, producing output $a = 0$. Both types have a reservation wage of 1 and are risk neutral. At a cost of 2 to himself and 1 to the job applicant, an employer can test a graduate and discover his true ability. Employers compete with each other in offering wages but they cooperate in revealing test results, so an employer knows if an applicant has already been tested and failed. There is just one period of work. The employer cannot commit to testing every applicant or any fixed percentage of them.

(a) Why is there no equilibrium in which either untalented workers do not apply or the employer tests every applicant?

Answer. If no untalented workers apply, the employer would deviate and save 2 by skipping the test and just hiring everybody who applies. Then the untalented workers would start to apply. If the employer tests every applicant, however, and pays only w_H, then no untalented worker will apply. Again, the employer would deviate and skip the test.

(b) In equilibrium, the employer tests workers with probability γ and pays those who pass the test w, the talented workers all present themselves for testing, and the untalented workers present themselves with probability α, where possibly $\gamma = 1$ or $\alpha = 1$. Find an expression for the equilibrium value of α in terms of w. Explain why α is not directly a function of x in this expression, even though the employer's main concern is that some workers have a productivity advantage of x.

Answer. Using the payoff-equating method of calculating a mixed strategy, and remembering that one must equate player 1's payoffs to find player 2's mixing probability, we must focus on the employer's profits. In the mixed-strategy equilibrium, the employer's profits are the same whether it tests a particular worker or not. Fraction $0.5 + 0.5\alpha$ of the workers will take the test, and the employer's cost for each one that applies is 2, whether he is hired or not, so

$$\begin{aligned} \pi(test) &= \left(\tfrac{0.5}{0.5+0.5\alpha}\right)(x-w) - 2 \\ &= \pi(no\ test) = \left(\tfrac{0.5}{0.5+0.5\alpha}\right)(x-w) + \left(\tfrac{0.5\alpha}{0.5+0.5\alpha}\right)(0-w), \end{aligned} \tag{1}$$

which yields

$$\alpha = \frac{2}{w-2}. \tag{2}$$

The naive answer to why expression (2) does not depend on x is that α is the worker's strategy, so there is no reason why it should depend on a parameter that enters only into the employer's payoffs. That is wrong, because usually in mixed strategy equilibria that is precisely the case, because the worker is choosing his probability in a way that makes the employer indifferent between his payoffs. Rather, what is going on here is that a talented worker's productivity is irrelevant to the decision of whether to test or not. The employer already knows he will hire all the talented workers, and the question for him in deciding whether to test is how costly it is to test and how costly it is to hire untalented workers.

(c) If $x = 9$, what are the equilibrium values of α, γ, and w?

Answer. We already have an expression for α from part (b). The next step is to find the the wage. Profits are zero in equilibrium, which requires that

$$\pi(no\ test) = \left(\frac{0.5}{0.5 + 0.5\alpha}\right)(x - w) + \left(\frac{0.5\alpha}{0.5 + 0.5\alpha}\right)(0 - w) = 0. \tag{3}$$

This implies that

$$\alpha = \frac{x - w}{w}. \tag{4}$$

Solving (2) and (3) together yields $\frac{2}{w-2} = \frac{x-w}{w}$, so

$$2w = (w - 2)(x - w) \tag{5}$$

Substituting $x = 9$ and solving equation (5) for w yields

$w^* = 6$ and $\alpha^* = \frac{9-6}{6} = .5$.

There is also a root $w = 3$ to equation (5), but it would violate an implicit assumption: that $\alpha \leq 1$, since it would make $\alpha = \frac{9-3}{3} = 2$.

We still need to find γ^*. In the mixed-strategy equilibrium, the untalented worker's profits are the same whether he applies or not, so

$$\pi(apply) = \gamma(-1 + 1) + (1 - \gamma)(-1 + w) = \pi(not\ apply) = 1. \tag{6}$$

Substituting $w = 6$ and solving for γ yields $(1 - \gamma)(-1 + 6) = 1$, so $(1 - \gamma) = .2$ and $\gamma^* = .8$.

(d) If $x = 8$, what are the equilibrium values of α, γ, and w?

Answer. Substituting $x = 8$ and solving equation (5) for w yields $w^* = 4$ and $\alpha^* = \frac{8-4}{4} = 1$. Thus, now all the untalented workers apply in equilibrium.

Now let us find γ^*. We need to make all the untalented workers want to apply, so we need

$$\pi(apply) = \gamma(-1 + 1) + (1 - \gamma)(-1 + w) \geq \pi(not\ apply) = 1. \tag{7}$$

Making equation (7) an equality, substituting $w = 6$ and solving for γ yields $(1 - \gamma)(-1 + 4) = 1$, so $(1 - \gamma) = 1/3$ and

$\gamma^* \leq 2/3$.

There is not a single equilibrium when $x = 8$, because the employer is indifferent over all values of γ (that is how we calculated α and w), and values over the entire range $\gamma \in [0, 2/3]$ will induce all the untalented workers to apply.

9.5. Insurance and State-Space Diagrams

Two types of risk-averse people, clean-living and dissolute, would like to buy health insurance. Clean-living people become sick with probability 0.3, and dissolute people with probability 0.9. In state-space diagrams with the person's wealth if he is healthy on the vertical axis and if he is sick on the horizontal, every person's initial endowment is (5,10), because his initial wealth is 10 and the cost of medical treatment is 5.

(a) What is the expected wealth of each type of person?

Answer. $E(W_c) = 8.5 (= 0.7(10) + 0.3(5))$. $E(W_d) = 5.5 (= 0.1(10) + 0.9(5))$.

(b) Draw a state-space diagram with the indifference curves for a risk- neutral insurance company that insures each type of person separately. Draw in the post-insurance allocations C_1 for the dissolute and C_2 for the clean- living under the assumption that a person's type is contractible.

Answer. See Figure A9.1.

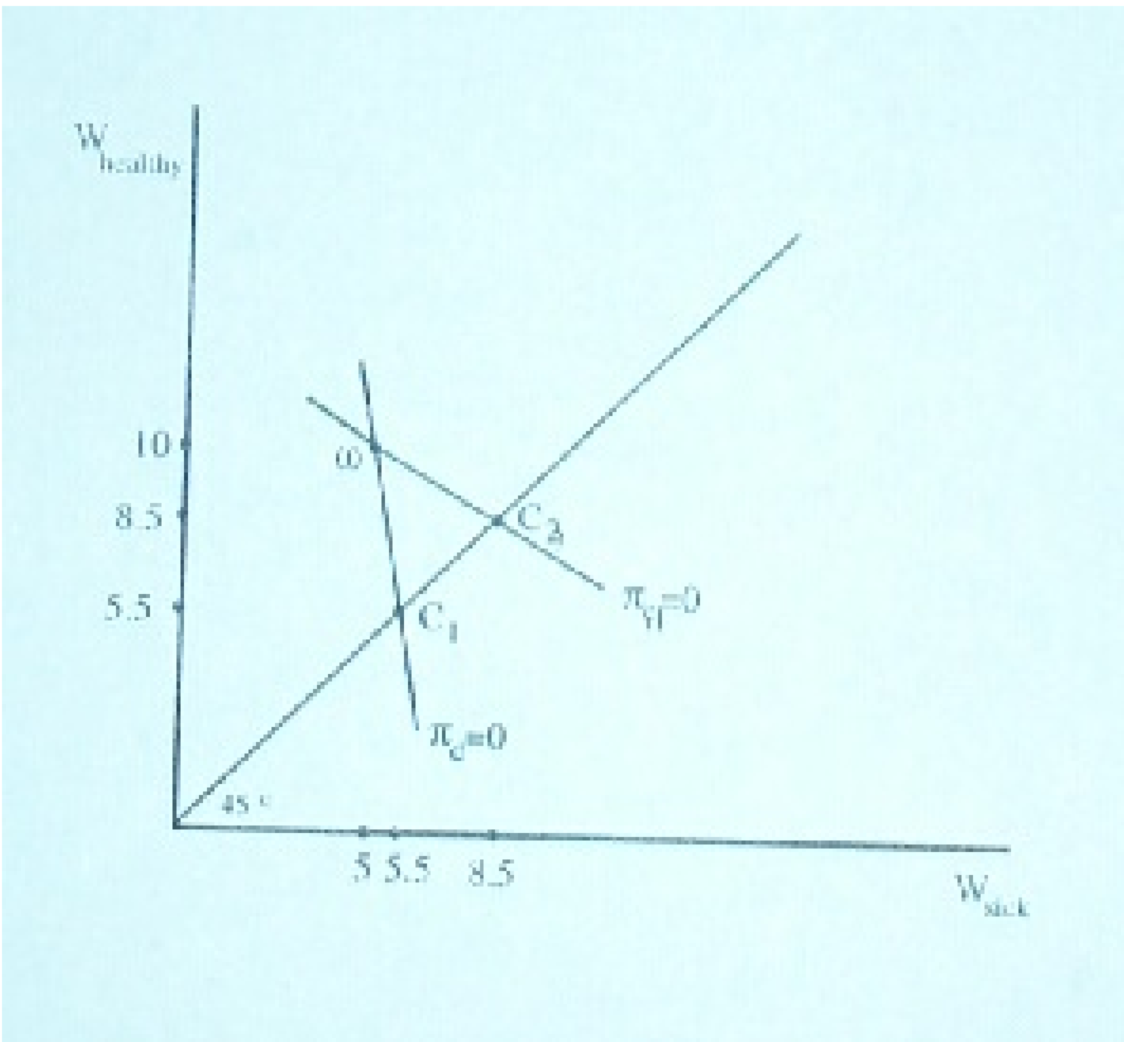

Figure A9.1: A State-Space Diagram Showing Indifference Curves for the Insurance Company

(c) Draw a new state-space diagram with the initial endowment and the indifference curves for the two types of people that go through that point.

Answer. See Figure A9.2

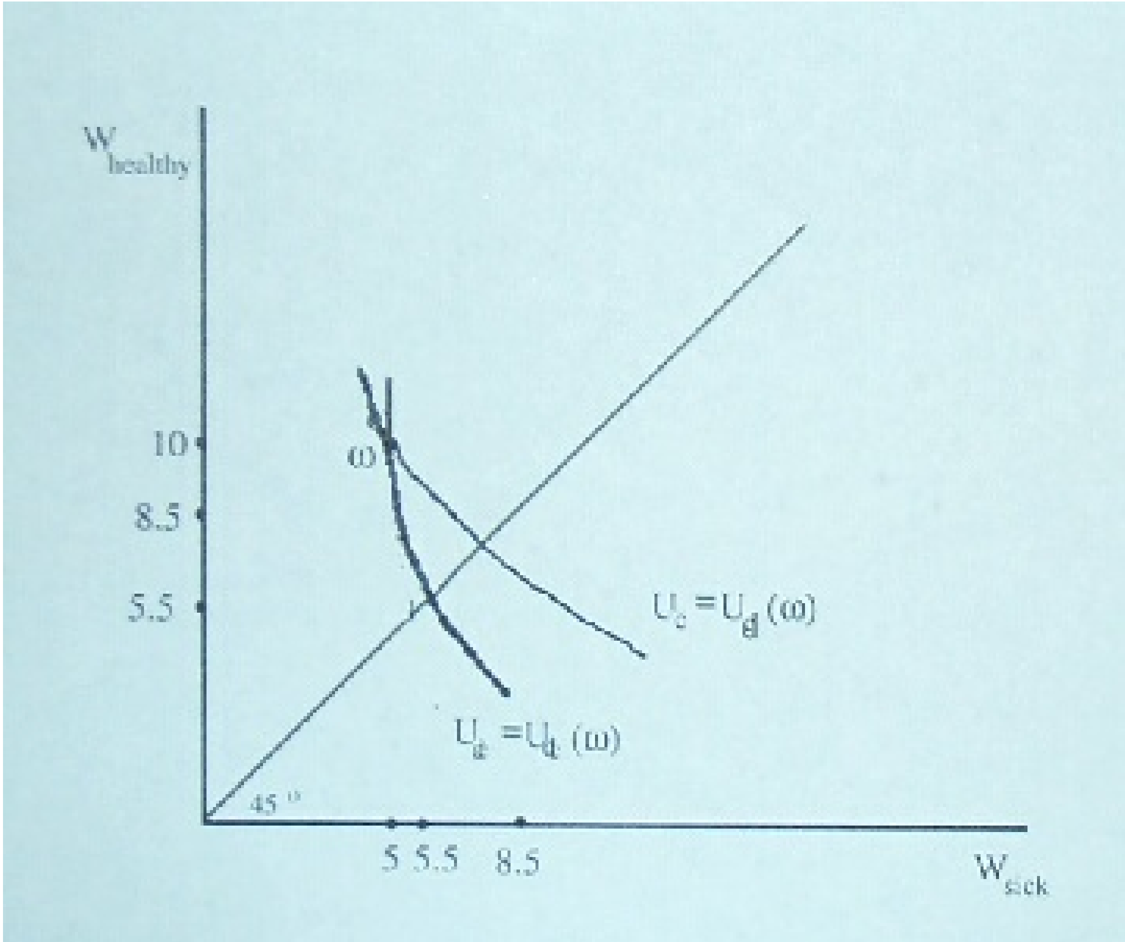

Figure A9.2: A State-Space Diagram Showing Indifference Curves for the Customers

(d) Explain why, under asymmetric information, no pooling contract C_3 can be part of a Nash equilibrium.

Answer. Call the pooling contract C_3. Because indifference curves for the the clean-living are flatter than for the dissipated, a contract C_4 can be found which yields positive profits because it attracts the clean-living but not the dissipated. See Figure A9.3.

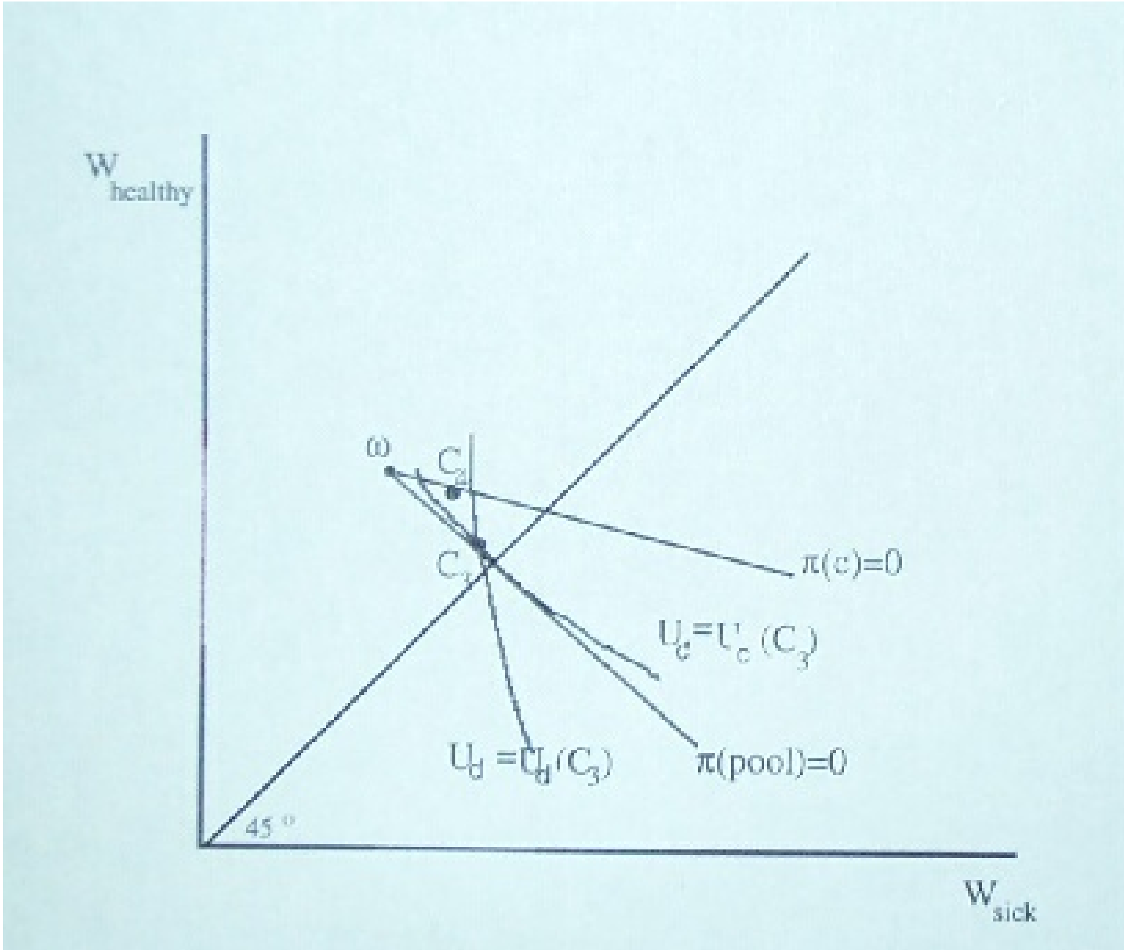

Figure A9.3: Why A Pooling Contract Cannot be Part of an Equilibrium

(e) If the insurance company is a monopoly, can a pooling contract be part of a Nash equilibrium?

<u>*Answer.*</u> Yes. If the insurance company is a monopoly, then a pooling contract can be part of a Nash equilibrium, because there is no other player who might deviate by offering a cream- skimming contract.