ODD

Answers to Odd-Numbered Problems, 4th Edition of Games and Information, Rasmusen

PROBLEMS FOR CHAPTER 13: Auctions

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This appendix contains answers to the odd-numbered problems in the fourth edition of *Games and Information* by Eric Rasmusen, which came out in 2006. The answers to the even-numbered problems are available to instructors or self-studiers on request to me at Erasmuse@indiana.edu.

13.1. Rent-Seeking

Two risk-neutral neighbors in 16th century England, Smith and Jones, have gone to court and are considering bribing a judge. Each of them makes a gift, and the one whose gift is the largest is awarded property worth £2,000. If both bribe the same amount, the chances are 50 percent for each of them to win the lawsuit. Gifts must be either £0, £900, or £2, 000.

(a) What is the unique pure-strategy equilibrium for this game?

<u>*Answer.*</u> Each bids £900, for expected profits of 100 each (=-900 + 0.5(2000)). This is an all-pay auction, but with restrictions on the bid amounts. Table A13.1 shows the payoffs (but also includes the payoffs for when the strategy of a bid of 1,500 is allowed). A player who deviates to 0 has a payoff of 0; a player who deviates to 2,000 has a payoff of 0. (0,0) is not an equilibrium, because the expected payoff is 1,000, but a player who deviated to 900 would have a payoff of 1,100.

Table A13.1: Bribes I

		Jones			
		£0	£900	£1500	£2000
	£0	1000,1000	0,1100	0,500	0,0
Smith:	£900	1100,0	100,100	-900, 500	- 900,0
	£1500	500,0	$500, -900$	$-500, -500$	$-1500, 0$
	£2000	0,0	$0, -900$	$0, -1500$	$-1000, -1000$

Payoffs to: (Smith, Jones).

(b) Suppose that it is also possible to give a £1500 gift. Why does there no longer exist a pure-strategy equilibrium?

<u>*Answer.*</u> If one player bids 0 or 900, the other would bid 1500, so we know 0 and 900 would not be used in equilibrium. If both player bid 1500, payoffs would be negative (= 2000/2-1500 each), so one could deviate to 0 and increase his payoff. If both bid 2000, one can profit by deviating to 0. If one player bids 1500 and the other bids 2000, the one with the bid of 1500 loses, for a payoff of -1500, and would be better off deviating to 0. This exhausts all the possibilities.

(c) What is the symmetric mixed-strategy equilibrium for the expanded game? What is the judge's expected payoff?

<u>*Answer.*</u> Let (θ_0, θ_{900}, $\theta_{1,500}$,$\theta_{2,000}$) be the probabilities. It is pointless ever to bid 2,000, because it can only yield zero or negative profits, so $\theta_{2,000} = 0$. In a symmetric mixed-strategy equilibrium, the return to the pure strategies is equal and the probabilities

add up to one, so

$$
\begin{aligned}
&\pi_{Smith}(0) = \pi_{Smith}(900) = \pi_{Smith}(1500) \\
&0.5\theta_0(2000) = -900 + \theta_0(2000) + 0.5\theta_{900}(2000) \\
&= -1500 + \theta_0(2000) + \theta_{900}(2000) + 0.5\theta_{1500}(2000),
\end{aligned} \tag{1}
$$

and

$$\theta_0 + \theta_{900} + \theta_{1500} = 1. \tag{2}$$

Solving out these three equations for three unknowns, the equilibrium is $(0.4, 0.5, 0.1, 0.0)$.

The judge's expected payoff is 1,200 $(= 2(0.5(900) + 0.1(1500)) = 2[450 + 150]))$.

Note: The results are sensitive to the bids allowed. Can you speculate as to what might happen if the strategy space were the whole continuum from 0 to 2000?

(d) In the expanded game, if the losing litigant gets back his gift, what are the two equilibria? Would the judge prefer this rule?

Answer. Table A13.2 shows the new outcome matrix. There are three equilibria: $x_1 = (900, 900), x_2 = 1500, 1500)$, and $x_3 = (2000, 2000)$.

Table A13.2: Bribes II

		Jones			
		£0	£900	£1500	£2000
	£0	1000,1000	0,1100	0,500	0,0
Smith:	£900	1100,0	**250,250**	0, 500	0,0
	£1500	500,0	500, 0	**250, 250**	0, 0
	£2000	0,0	0, 0	0, 0	**0, 0**

Payoffs to: (Smith, Jones).

The judge's payoff was 1200 under the unique mixed-strategy equilibrium in the original game. Now, his payoff is either 900, 1500, or 2000. Thus, whether he prefers the new rules depends on which equilibrium is played out in it.

13.3. Government and Monopoly (medium)

Incumbent Apex and potential entrant Brydox are bidding for government favors in the widget market. Apex wants to defeat a bill that would require it to share its widget patent rights with Brydox. Brydox wants the bill to pass. Whoever offers the chairman of the House Telecommunications Committee more campaign contributions wins, and the loser pays nothing. The market demand curve for widgets is $P = 25 - Q$, and marginal cost is constant at 1.

(a) Who will bid higher if duopolists follow Bertrand behavior? How much will the winner bid?

Answer. Apex bids higher, because it gets monopoly profits from winning, and Bertrand profits equal zero. Apex can bid some small ϵ and win.

(b) Who will bid higher if duopolists follow Cournot behavior? How much will the winner bid?

Answer. Monopoly profits are found from the problem

$$\underset{Q_a}{Maximize} \qquad Q_a(25 - Q_a - 1), \tag{3}$$

which has the first order condition $25 - 2Q_a - 1 = 0$, so that $Q_a = 12$ and $\pi_a = 144$ $(= 12(25 - 12 - 1))$.

Apex's Cournot duopoly profit is found by solving the problem

$$\underset{Q_a}{Maximize} \qquad Q_a(25 - [Q_a + Q_b] - 1), \tag{4}$$

which has the first order condition $25 - 2Q_a - Q_b - 1 = 0$, so that if the equilibrium is symmetric and $Q_b = Q_a$, then $Q_a = 8$ and $\pi_a = 64$ $(= 8(25 - [8 + 8] - 1))$.

Brydox will bid up to 64, since that is its gain from being a duopolist rather than out of the industry altogether. Apex will bid up to $80(= 144 - 64)$, and so will win the auction at a price of 64.

(c) What happens under Cournot behavior if Apex can commit to giving away its patent freely to everyone in the world if the entry bill passes? How much will Apex bid?

Answer. Apex will bid some small ϵ and win. It will commit to giving away its patent if the bill succeeds, which means that if the bill succeeds, the industry will have zero profits and Brydox has no incentive to bid a positive amount to secure entry.

13.5. A Teapot Auction with Incomplete Information

Smith believes that Brown's value v_b for a teapot being sold at auction is 0 or 100 with equal probability. Smith's value of $v_s = 400$ is known by both players.

(a) What are the players' equilibrium strategies in an open cry auction? You may assume that in case of ties, Smith wins the auction.

Answer. Brown bids up to his value of 0 or 100. Smith bids up to his value of 400. Thus, Smith wins, at a price of 0 or of 100.

(b) What are the players' equilibrium strategies in a first-price sealed-bid auction? You may assume that in case of ties, Smith wins the auction.

Answer. Brown bids either 0 or 100 in equilibrium. Smith always bids 100, because his value is so high that winning is more important than paying a low price.

(c) Now let $v_s = 102$ instead of 400. Will Smith use a pure strategy? Will Brown? You need not find the exact strategies used.

Answer. Smith would use a mixed strategy, and while Brown would still offer 0 if his value were 0, if his value were 100 he would use a mixed strategy too. No pure strategy can be part of a Nash equilibrium, because if Smith always bid a value $x < 100$, Brown would always bid $x + \varepsilon$, in which case Smith would deviate to $x + \varepsilon$, and if Smith bid $x \geq 100$ he would be paying 100 more than necessary half the time.