ODD

Answers to Odd-Numbered Problems, 4th Edition of Games and Information, Rasmusen

PROBLEMS FOR CHAPTER 14: Pricing

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This appendix contains answers to the odd-numbered problems in the fourth edition of *Games and Information* by Eric Rasmusen, which I am working on now and perhaps will come out in 2006. The answers to the even- numbered problems are available to instructors or self-studiers on request to me at Erasmuse@indiana.edu.

Other books which contain exercises with answers include Bierman & Fernandez (1993), Binmore (1992), Fudenberg & Tirole (1991a), J. Hirshleifer & Riley (1992), Moulin (1986), and Gintis (2000). I must ask pardon of any authors from whom I have borrowed without attribution in the problems below; these are the descendants of problems that I wrote for teaching without careful attention to my sources.

14.1. Differentiated Bertrand with Advertising

Two firms that produce substitutes are competing with demand curves

$$q_1 = 10 - \alpha p_1 + \beta p_2 \tag{1}$$

and

$$q_2 = 10 - \alpha p_2 + \beta p_1. \tag{2}$$

Marginal cost is constant at $c = 3$. A player's strategy is his price. Assume that $\alpha > \beta/2$.

(a) What is the reaction function for Firm 1? Draw the reaction curves for both firms.

<u>*Answer.*</u> Firm 1's profit function is

$$\pi_1 = (p_1 - c)q_1 = (p_1 - 3)(10 - \alpha p_1 + \beta p_2). \tag{3}$$

Differentiating with respect to p_1 and solving the first order condition gives the reaction function

$$p_1 = \frac{10 + \beta p_2 + 3\alpha}{2\alpha}. \tag{4}$$

This is shown in Figure A14.1.

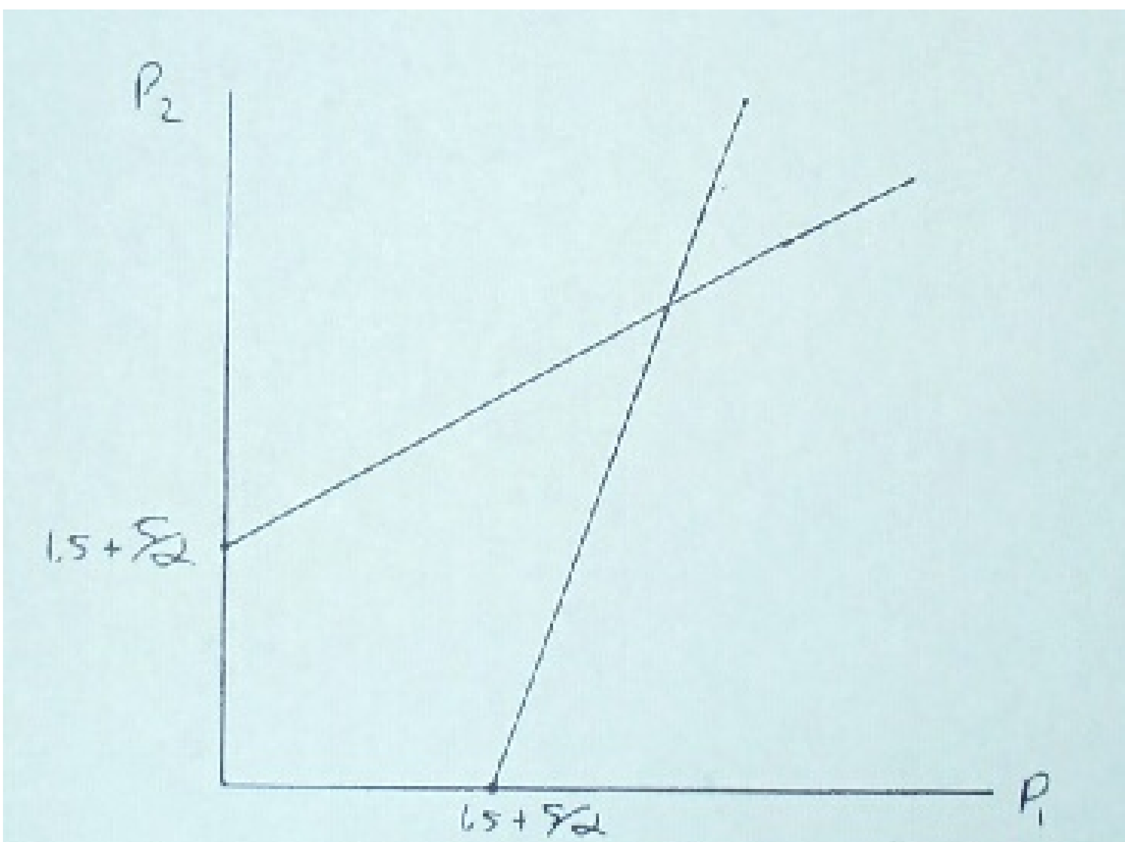

Figure A14.1: The Reaction Curves in a Bertrand Game with Advertising

(b) What is the equilibrium? What is the equilibrium quantity for Firm 1?

<u>Answer.</u> Using the symmetry of the problem, set $p_1 = p_2$ in the reaction function for Firm 1 and solve, to give $p_1^* = p_2^* = \frac{10+3\alpha}{2\alpha-\beta}$. Using the demand function for Firm 1, $q_1 = \frac{10\alpha+3\alpha(\beta-\alpha)}{2\alpha-\beta}$.

(c) Show how Firm 2's reaction function changes when β increases. What happens to the reaction curves in the diagram?

<u>Answer.</u> The slope of Firm 2's reaction curve is $\frac{\partial p_2}{\partial p_1} = \frac{\beta}{2\alpha}$. The change in this when β changes is $\frac{\partial^2 p_2}{\partial p_1 \partial \beta} = \frac{1}{2\alpha} > 0$. Thus, Firm 2's reaction curve becomes steeper, as shown in Figure A14.2.

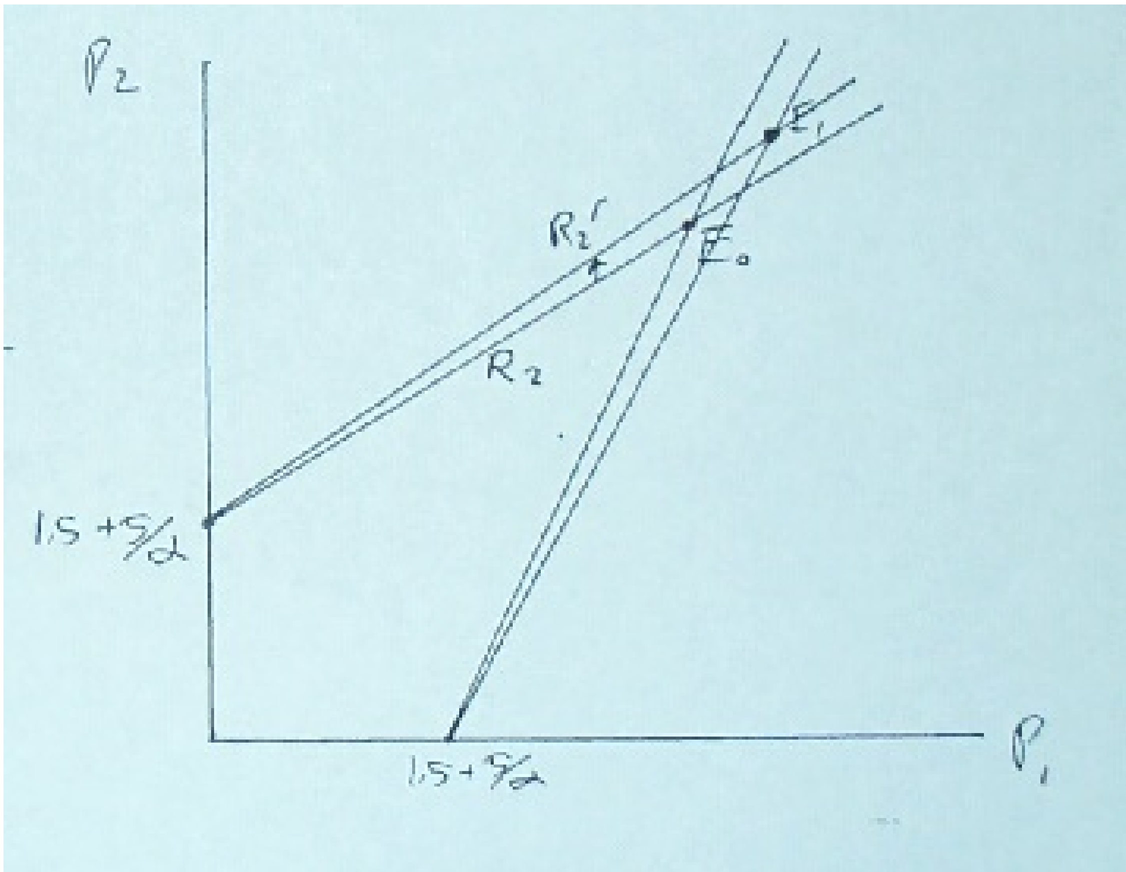

Figure A14.2: How Reaction Curves Change When β Increases

(d) Suppose that an advertising campaign could increase the value of β by one, and that this would increase the profits of each firm by more than the cost of the campaign. What does this mean? If either firm could pay for this campaign, what game would result between them?

<u>Answer.</u> The meaning of an increase in β is that a firm's quantity demanded becomes more responsive to the other firm's price, if it charges a high price. The meaning is really mixed: partly, the goods become closer substitutes, and partly, total demand for the two goods increases.

If either firm could pay, then a game of "Chicken" results, with payoffs something like in Table A14.1, where the ad campaign costs 1 and yields extra profits of B to each firm.

Table A14.1: An Advertising Chicken Game

		Firm 2		
		Advertise		*Do not advertise*
	Advertise	B-1,B-1	$\rightarrow$	**B-1,B**
Firm 1:		$\downarrow$		$\uparrow$
	Do not advertise	**B,B-1**	$\leftarrow$	0,0

Payoffs to: (Firm 1, Firm 2).

14.3. Differentiated Bertrand

Two firms that produce substitutes have the demand curves

$$q_1 = 1 - \alpha p_1 + \beta(p_2 - p_1) \tag{5}$$

and

$$q_2 = 1 - \alpha p_2 + \beta(p_1 - p_2), \tag{6}$$

where $\alpha > \beta$. Marginal cost is constant at c, where $c < 1/\alpha$. A player's strategy is his price.

(a) What are the equations for the reaction curves $p_1(p_2)$ and $p_2(p_1)$? Draw them.

<u>*Answer.*</u> Firm 1 solves the problem of maximizing $\pi_1 = (p_1 - c)q_1 = (p_1 - c)(1 - \alpha p_1 + \beta[p_2 - p_1])$ by choice of p_1 . The first order condition is $1 - 2(\alpha+\beta)p_1 + \beta p_2 + (\alpha+\beta)c = 0$, which gives the reaction function $p_1 = \frac{1+\beta p_2+(\alpha+\beta)c}{2(\alpha+\beta)}$. For p_2: $p_2 = \frac{1+\beta p_1+(\alpha+\beta)c}{2(\alpha+\beta)}$. Figure A14.3 shows the reaction curves. Note that $\beta > 0$, because the goods are substitutes.

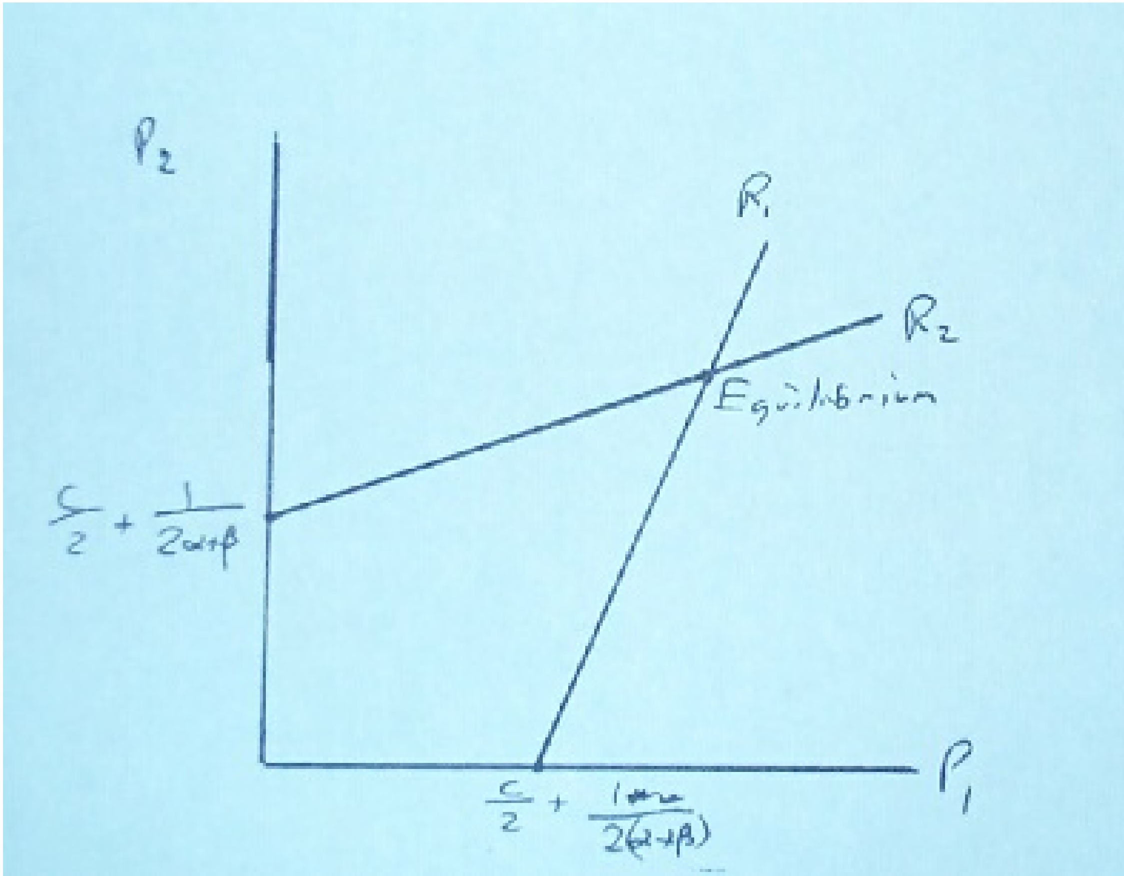

Figure A14.3: Reaction Curves for the Differentiated Bertrand Game

(b) What is the pure-strategy equilibrium for this game?

<u>*Answer.*</u> This game is symmetric, so we can guess that $p_1^* = p_2^*$. In that case, using the reaction curves, $p_1^* = p_2^* = \frac{1+(\alpha+\beta)c}{2\alpha+\beta}$.

(c) What happens to prices if α, β, or c increase?

<u>*Answer.*</u> The response of p^* to an increase in α is:

$$\frac{\partial p^*}{\partial \alpha} = \frac{c}{2\alpha+\beta} - \frac{2[1+(\alpha+\beta)c]}{(2\alpha+\beta)^2} = \left(\frac{1}{(2\alpha+\beta)^2}\right)(2\alpha c + \beta c - 2 - 2\alpha c - 2\beta c) < 0. \tag{7}$$

The derivative has the same sign as $-\beta c - 2 < 0$, so, since $\beta > 0$, the price falls as α rises. This makes sense— α represents the responsiveness of the quantity demanded to the firm's own price.

The increase in p^* when β increases is:

$$\frac{\partial p^*}{\partial \beta} = \frac{c}{(2\alpha+\beta)} - \frac{1+(\alpha+\beta)c}{(2\alpha+\beta)^2} = \left(\frac{1}{(2\alpha+\beta)^2}\right)(2\alpha c + \beta c - 1 - \alpha c - \beta c) < 0. \tag{8}$$

The price falls with β, because $c < 1/\alpha$.

The increase in p^* when c increases is:

$$\frac{\partial p^*}{\partial c} = \frac{\alpha+\beta}{2\alpha+\beta} > 0. \tag{9}$$

When the marginal cost rises, so does the price.

(d) What happens to each firm's price if α increases, but only Firm 2 realizes it (and Firm 2 knows that Firm 1 is uninformed)? Would Firm 2 reveal the change to Firm 1?

<u>*Answer.*</u> From the equation for the reaction curve of Firm 1, it can be seen that the reaction curve will shift and swivel as in Figure A.13. This is because $\frac{\partial p_2}{\partial p_1} = \frac{\beta}{2(\alpha+\beta)}$, so $\frac{\partial^2 p_2}{\partial p_1 \partial \beta} = -\frac{\beta}{2(\alpha+\beta)^2} < 0$. Firm 2's reaction curve does not change, and it believes that Firm 1's reaction curve has not changed either, so Firm 2 has no reason to change its price. The equilibrium changes from E_0 to E_1: Firm 1 maintains its price, but Firm 2 reduces its price. Firm 2 would not want to reveal the change to Firm 1, because then Firm 1 would also reduce its price (and Firm 2 would reduce its price still further), and the new equilibrium would be E_2.

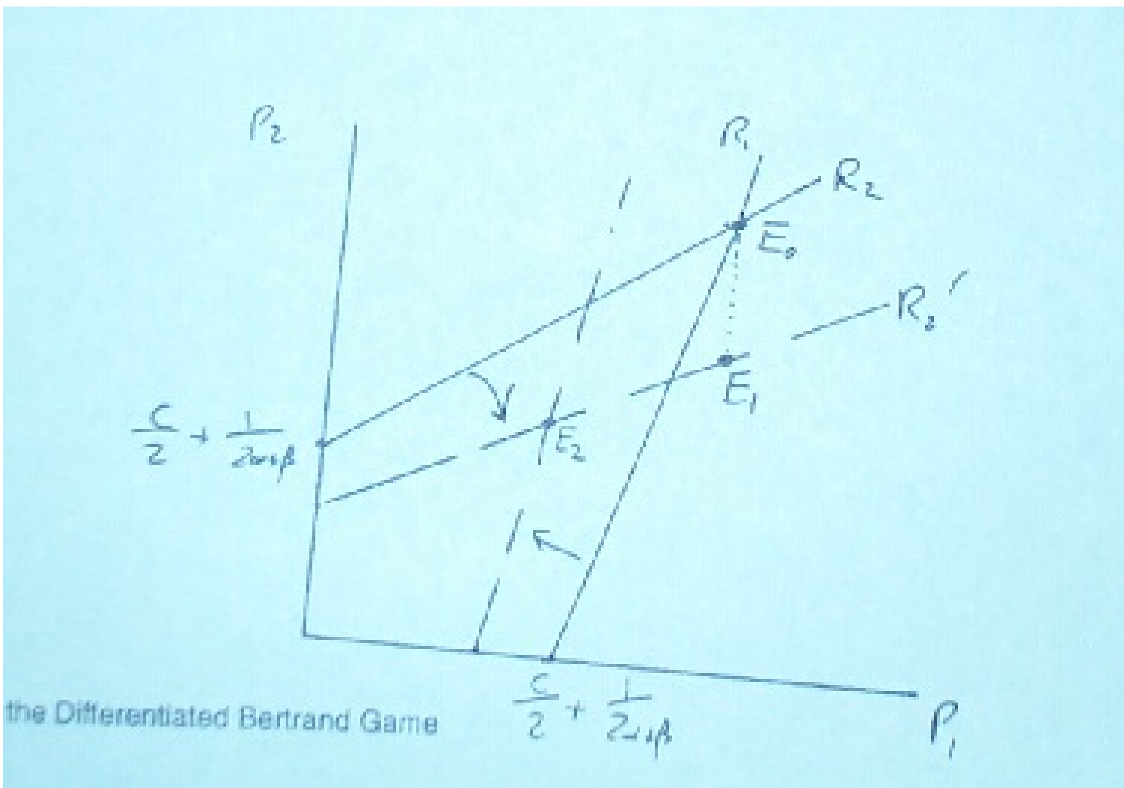

Figure A14.4: Changes in the Reaction Curves

14.5. Price Discrimination

A seller faces a large number of buyers whose market demand is given by $P = \alpha - \beta Q$. Production marginal cost is constant at c.

(a) What is the monopoly price and profit?

Answer. Profit is $PQ - cQ$ or $(\alpha - \beta Q - c)Q$. The first order condition is $\alpha - 2\beta Q - c = 0$, so $Q = \frac{\alpha - c}{2\beta}$. The price is then $P = \alpha - \beta\frac{\alpha-c}{2\beta} = \alpha - \frac{\alpha-c}{2} = \frac{\alpha+c}{2}$. The profit is $(P-c)Q = (\frac{\alpha+c}{2} - c)\frac{\alpha-c}{2\beta} = \frac{(\alpha-c)^2}{4\beta}$.

(b) What are the prices under perfect price discrimination if the seller can make take-it-or-leave-it offers? What is the profit?

Answer. Under perfect price discrimination, there is a continuum of prices along the demand curve from α to c. The profit equals the area of the triangle under the demand curve and above the flat MC curve, which is $(1/2)(\alpha - c)Q(c) = (1/2)(\alpha - c)\frac{\alpha-c}{\beta} = \frac{(\alpha-c)^2}{2\beta}$. Notice how profit has doubled compared to the simple monopoly profit.

(c) What are the prices under perfect price discrimination if the buyer and sellers bargain over the price and split the surplus evenly? What is the profit?

Answer. If buyers and sellers split the surplus evenly, then instead of the seller getting the entire surplus, he only gets half, so profits are half those found in part (b). There is a continuum of prices between $c + \frac{\alpha-c}{2}$ and c. The profit is $\frac{(\alpha-c)^2}{4\beta}$, the same as the monopoly profit in this special case.