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Answers to Odd-Numbered Problems, 4th Edition of Games and Information, Rasmusen

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PROBLEMS FOR CHAPTER 5 Reputation and Repeated Games

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This appendix contains answers to the odd-numbered problems in the gourth edition of *Games and Information* by Eric Rasmusen, which appeared in 2006. Other books which contain exercises with answers include Bierman & Fernandez (1993), Binmore (1992), Fudenberg & Tirole (1991a), J. Hirshleifer & Riley (1992), Moulin (1986), and Gintis (2000). I ask pardon of any authors from whom I have borrowed without attribution in the problems below; these are the descendants of problems that I wrote for teaching without careful attention to my sources.

PROBLEMS FOR CHAPTER 5 Reputation and Repeated Games

5.1. Overlapping Generations (Samuelson [1958])
There is a long sequence of players. One player is born in each period t, and he lives for periods t and $t+1$. Thus, two players are alive in any one period, a youngster and an oldster. Each player is born with one unit of chocolate, which cannot be stored. Utility is increasing in chocolate consumption, and a player is very unhappy if he consumes less than 0.3 units of chocolate in a period: the per-period utility functions are $U(C) = -1$ for $C < 0.3$ and $U(C) = C$ for $C \geq 0.3$, where C is consumption. Players can give away their chocolate, but, since chocolate is the only good, they cannot sell it. A player's action is to consume X units of chocolate as a youngster and give away $1 - X$ to some oldster. Every person's actions in the previous period are common knowledge, and so can be used to condition strategies upon.

(a) If there is finite number of generations, what is the unique Nash equilibrium?

<u>*Answer.*</u> X=1. The Chainstore Paradox applies. Youngster T, the last one, has no incentive to give anything to Oldster $T-1$. Therefore, Youngster $T-1$ has no incentive either, and so for for every t.

(b) If there are an infinite number of generations, what are two Pareto- ranked perfect equilibria?

<u>*Answer.*</u> (i) *($X = 1$, regardless of what others do)*, and (ii) *($X = 0.5$, unless some player has deviated, in which case $X = 1$)*. Equilibrium (ii) is pareto superior.

(c) If there is a probability θ at the end of each period (after consumption takes place) that barbarians will invade and steal all the chocolate (leaving the civilized people with payoffs of -1 for any X), what is the highest value of θ that still allows for an equilibrium with $X = 0.5$?

<u>*Answer.*</u> The payoff from the equilibrium strategy is $0.5+(1-\theta)0.5+\theta(-1) = 1-1.5\theta$. The payoff from deviating to $X = 1$ is $1 - 1 = 0$. These are equal if $1 - 1.5\theta = 0$; that is, if $\theta = \frac{2}{3}$. Hence, θ can take values up to $\frac{2}{3}$ and the $X = 0.5$ equilibrium can still be maintained.

5.3. Repeated Games (see Benoit & Krishna [1985])
Players Benoit and Krishna repeat the game in Table 7 three times, with discounting:

Table 7: A Benoit-Krishna Game

		Krishna		
		Deny	*Waffle*	*Confess*
	Deny	10,10	$-1, -12$	$-1, 15$
Benoit:	*Waffle*	$-12, -1$	8,8	$-1, -1$
	Confess	$15, -1$	$-1,-1$	$0, 0$

Payoffs to: (Benoit, Krishna).

(a) Why is there no equilibrium in which the players play *Deny* in all three periods?

<u>Answer.</u> If Benoit and Krishna both chose *Deny* in the third period, Krishna would get a payoff of 10 in that period. He could increase his payoff by deviating to $Confess$.

(b) Describe a perfect equilibrium in which both players pick *Deny* in the first two periods.

<u>Answer.</u> In the last period, any equilibrium has to have the players either both choosing $Confess$ or both choosing $Waffle$ (which means to equivocate, to talk but neither to quite deny or quite confess). Consider the following proposed equilibrium behavior for each player:

1. Choose *Deny* in the first period.

2. Choose *Deny* in the second period unless someone chose a different action in the first period, in which case choose $Confess$.

3. Choose $Waffle$ in the third period unless someone chose something other than *Deny* in the first or second period, in which case choose $Confess$.

This is an equilibrium. In the third period, a deviator to either *Deny* or $Confess$ would have a payoff of -1 instead of 8 in that period. If, however, someone has already deviated in an earlier period, each player expects the other to choose $Confess$, in which case $Confess$ is his best response.

In the second period, if a player deviates to *Deny* he will have a payoff of 15 instead of 10 in that period. In the third period, however, his payoff will then be 0 instead of 8, because the actions will be $(Confess, Confess)$ instead of $(Waffle, Waffle)$. If the discount rate is low enough (for example $r = 0$), then deviation in the second period is not profitable. If some other player has deviated in the first period, however, the players expect each other to choose $Confess$ in the second period and that is self- confirming.

In the first period, if a player deviates to *Deny* he will have a payoff of 15 instead of 10 in that period. In the second period, however, his payoff will then be 0 instead of 10, because the actions will be $(Confess, Confess)$ instead of $(Deny, Deny)$. And in the third period his payoff will then be 0 instead of 8, because the actions will be $(Confess, Confess)$ instead of $(Waffle, Waffle)$. If the discount rate is low enough (for example $r = 0$), then deviation in the first period is not profitable.

(c) Adapt your equilibrium to the twice-repeated game.

<u>*Answer.*</u> Simply leave out the middle period of the three-period model:

1. Choose *Deny* in the first period.

2. Choose *Waffle* in the second period unless someone chose something other than *Deny* in the first period, in which case choose *Confess*.

(d) Adapt your equilibrium to the T-repeated game.

<u>*Answer.*</u> Now we just add extra middle periods:

1. Choose *Deny* in the first period.

2. Choose *Deny* in the second period unless someone chose a different action in the first period, in which case choose *Confess*.

t. Choose *Deny* in the t'th period for $t = 3, ..., T-1$ unless someone chose a different action previously, in which case choose *Confess*.

T. Choose *Waffle* in the third period unless someone chose something other than *Deny* previously, in which case choose *Confess*.

(e) What is the greatest discount rate for which your equilibrium still works in the 3-period game?

<u>*Answer.*</u> It is harder to prevent deviation in the second period than in the first period, because deviation in the first period leads to lower payoffs in two future periods instead of one. So if a discount rate is low enough to prevent deviation in the second period, it is low enough to prevent deviation in the first period.

The equilibrium payoff in the subgame starting with the second period is, if the discount rate is ρ,

$$10 + \frac{1}{1+\rho}(8)$$

The payoff to deviating to *Confess* in the second period and then choosing *Confess* in the third period is

$$15 + \frac{1}{1+\rho}(0).$$

Equating these two payoffs yields $10 + \frac{8}{1+\rho} = 15$, so $8 = 5(1+\rho)$, $3 = 5\rho$, and $\rho = .6$. This is the greatest discount rate for which the strategy combination in part (a) remains an equilibrium.

5.5. The Repeated Prisoner's Dilemma

Set $P = 0$ in the general Prisoner's Dilemma in Table 1.10 of Chapter 1, and assume that $2R > S + T$.

(a) Show that the Grim Strategy, when played by both players, is a perfect equilibrium for the infinitely repeated game. What is the maximum discount rate for which the Grim Strategy remains an equilibrium?

<u>*Answer.*</u> The grim strategy is a perfect equilibrium because the payoff from continued cooperation is $R + \frac{R}{r}$, which for low discount rates is greater than the payoff from $(Confess, Deny)$ once and $(Confess, Confess)$ forever after, which is $T + \frac{0}{r}$. To find the maximum discount rate, equate these two payoffs: $R + \frac{R}{r} = T$. This means that $r = \frac{R}{T-R}$ is the maximum.

(b) Show that Tit-for-Tat is not a perfect equilibrium in the infinitely repeated Prisoner's Dilemma with no discounting.[1]

<u>*Answer.*</u> Suppose Row has played $Confess$. Will Column retaliate? If both follow tit-for-tat after the deviation, retaliation results in a cycle of $(Confess, Deny)$, $(Deny, Confess)$, forever. Row's payoff is $T+S+T+S+....$ If Column forgives, and they go back to cooperating, on the other hand, his payoff is $R+R+R+R+....$ Comparing the first four periods, forgiveness has the higher payoff because $4R > 2S + 2T$. The payoffs of the first four periods simply repeat an infinite number of times to give the total payoff, so forgiveness dominates retaliation, and tit-for-tat is not perfect.

Kalai, Samet & Stanford (1988) pointed this out.

5.7. Grab the Dollar

Table 5.10 shows the payoffs for the simultaneous-move game of Grab the Dollar. A silver dollar is put on the table between Smith and Jones. If one grabs it, he keeps the dollar, for a payoff of 4 utils. If both grab, then neither gets the dollar, and both feel bitter. If neither grabs, each gets to keep something.

Table 5.10: Grab the Dollar

		Jones	
		Grab (θ)	*Wait* $(1-\theta)$
Smith:	*Grab* (θ)	$-1, -1$	$4, 0$
	Wait $(1-\theta)$	$0, 4$	1,1

Payoffs to: (Smith, Jones).

(a) What are the evolutionarily stable strategies?

<u>*Answer.*</u> The ESS is mixed and unique. Let $Prob(Grab) = \theta$. Then $\pi(Grab) = -1(\theta) + 4(1-\theta) = \pi(Wait) = 0(\theta) + 1(1-\theta)$, which solves to $\theta = 3/4$. Three fourths of the population plays *Grab*.

[1] xxx Add: The idea is informally explained on page 112).

(b) Suppose each player in the population is a point on a continuum, and that the initial amount of players is 1, evenly divided between *Grab* and *Wait*. Let $N_t(s)$ be the amount of players playing a particular strategy in period t and let $\pi_t(s)$ be the payoff. Let the population dynamics be $N_{t+1}(i) = (2N_t(i))\left(\frac{\pi_t(i)}{\sum_j \pi_t(j)}\right)$. Find the missing entries in Table 5.11.

Table 5.11: Grab the Dollar, Dynamics

t	$N_t(G)$	$N_t(W)$	$N_t(total)$	θ	$\pi_t(G)$	$\pi_t(w)$
0	0.5	0.5	1	0.5	1.5	0.5
1						
2						

<u>*Answer.*</u> See Table A5.1.

Table A5.1: Grab the Dollar, Dynamics I

t	$N_t(G)$	$N_t(W)$	$N_t(total)$	θ	$\pi_t(G)$	$\pi_t(w)$
0	0.5	0.5	1	0.5	1.5	0.5
1	0.75	0.25	1	0.75	0.25	0.25
2	0.75	0.25	1	0.75	0.25	0.25

(c) Repeat part (b), but with the dynamics $N_{t+t}(s) = [1 + \frac{\pi_t(s)}{\sum_j \pi_t(j)}][2N_t(s)]$.

<u>*Answer.*</u> See Table A5.2.

Table A5.2: Grab the Dollar, Dynamics II

t	$N_t(G)$	$N_t(W)$	$N_t(total)$	θ	$\pi_t(G)$	$\pi_t(w)$
0	.5	0.5	1	.5	1.5	0.5
1	1.75	1.25	3	0.58	1.1	0.42
2	6.03	3.19	9.22	0.65	0.75	0.35

(d) Which three games that have appeared so far in the book resemble *Grab the Dollar*?

<u>*Answer.*</u> Chicken, the Battle of the Sexes, and the Hawk-Dove Game.