# ODD

**Answers to Odd-Numbered Problems, 4th Edition of Games and Information, Rasmusen**

**PROBLEMS FOR CHAPTER 7: Moral Hazard: Hidden Actions**

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This appendix contains answers to the odd-numbered problems in the fourth edition of *Games and Information* by Eric Rasmusen. The answers to the even-numbered problems are available to instructors or self-studiers on request to me at Erasmuse@indiana.edu.

## PROBLEMS FOR CHAPTER 7: Moral Hazard: Hidden Actions

### 7.1. First-Best Solutions in a Principal-Agent Model

Suppose an agent has the utility function of $U = \sqrt{w} - e$, where $e$ can assume the levels 0 or 1. Let the reservation utility level be $\overline{U} = 3$. The principal is risk neutral. Denote the agent's wage, conditioned on output, as $\underline{w}$ if output is 0 and $\overline{w}$ if output is 100. Table 5 shows the outputs.

**Table 5: A Moral Hazard Game**

| **Effort** | **Probability of Output of** 0 | 100 | Total |
|---|---|---|---|
| *Low* ($e = 0$) | 0.3 | 0.7 | 1 |
| *High* ($e = 1$) | 0.1 | 0.9 | 1 |

(a) What would the agent's effort choice and utility be if he owned the firm?

<u>*Answer.*</u> The agent gets everything in this case. His utility is either

$$U(High) = 0.1(0) + 0.9\sqrt{100} - 1 = 8 \tag{1}$$

or

$$U(Low) = 0.3(0) + 0.7\sqrt{100} - 0 = 7. \tag{2}$$

So the agent chooses high effort and a utility of 8.

(b) If agents are scarce and principals compete for them, what will the agent's contract be under full information? His utility?

<u>*Answer.*</u> The efficient effort level is *High*, which produces an expected output of 90. The principal's profit is zero, because of competition. Since the agent is risk averse, he should be fully insured in equilibrium: $\overline{w} = \underline{w} = 90$ But he should get this only if his effort is high. Thus, the contract is w=90 if effort is high, w=0 if effort is low. The agent's utility is 8.5 $(= \sqrt{90} - 1$, rounded).

(c) If principals are scarce and agents compete to work for them, what would the contract be under full information? What will the agent's utility and the principal's profit be in this situation?

*Answer.* The efficient effort level is high. Since the agent is risk averse, he should be fully insured in equilibrium: $\overline{w} = \underline{w} = w$. The contract must satisfy a participation constraint for the agent, so $\sqrt{w} - 1 = 3$. This yields $w = 16$, and a utility of 3 for the agent. The actual contract specified a wage of 16 for high effort and 0 for low effort. This is incentive compatible, because the agent would get only 0 in utility if he took low effort. The principal's profit is 74 (= 90-16).

(d) Suppose that $U = w - e$. If principals are the scarce factor and agents compete to work for principals, what would the contract be when the principal cannot observe effort? (Negative wages are allowed.) What will be the agent's utility and the principal's profit be in this situation?

*Answer.* The contract must satisfy a participation constraint for the agent, so U = 3. Since effort is 1, the expected wage must equal 4. One way to produce this result is to allow the agent to keep all the output, plus 4 extra for his labor, but to make him pay the expected output of 90 for this privilege ("selling the store"). Let $\overline{w} = 14$ and $\underline{w} = -86$ (other contracts also work). Then expected utility is 3 ($= 0.1(-86) + 0.9(14) - 1 = -8.6 + 12.6 - 1$). Expected profit is 86 ($= 0.1(0 - -86) + 0.9(100 - 14) = 8.6 + 77.4$).

### 7.3. Why Entrepreneurs Sell Out

Suppose an agent has a utility function of $U = \sqrt{w} - e$, where $e$ can assume the levels 0 or 2.4, and his reservation utility is $\overline{U} = 7$. The principal is risk neutral. Denote the agent's wage, conditioned on output, as $w(0)$, $w(49)$, $w(100)$, or $w(225)$. Table 7.7 shows the output.

**Table 7: Entrepreneurs Selling Out**

| Method | Probability of Output of 0 | 49 | 100 | 225 | Total |
|---|---|---|---|---|---|
| *Safe* ($e = 0$) | 0.1 | 0.1 | 0.8 | 0 | 1 |
| *Risky* ($e = 2.4$) | 0 | 0.5 | 0 | 0.5 | 1 |

(a) What would the agent's effort choice and utility be if he owned the firm?

<u>*Answer.*</u> $U(safe) = 0 + 0.1\sqrt{49} + 0.8\sqrt{100} + 0 - 0 = 0.7 + 8 = 8.7$. $U(risky) = 0 + 0.5\sqrt{49} + 0.5\sqrt{225} - 2.4 = 3.5 + 7.5 - 2.4 = 8.6$. Therefore he will choose the safe method, e= 0, and utility is 8.7.

(b) If agents are scarce and principals compete for them, what will the agent's contract be under full information? His utility?

<u>*Answer.*</u> Agents are scarce, so $\pi = 0$. Since agents are risk averse, it is efficient to shield them from risk. If the risky method is chosen, then $w = 0.5(49) + 0.5(225) = 24.5 + 112.5 = 137$. Utility is 9.3 ($\sqrt{137} - 2.4 = 11.7 - 2.4$). If the safe method is chosen, then $w = 0.1(49) + 0.8(100) = 84.9$. Utility is $U = \sqrt{84.9} = 9.21$ . Therefore, the optimal contract specifies a wage of 137 if the risky method is used and 0 (or any wage less than 49) if the safe method is used. This is better for the agent than if he ran the firm by himself and used the safe method.

(c) If principals are scarce and agents compete to work for principals, what will the contract be under full information? What will the agent's utility and the principal's profit be in this situation?

<u>*Answer.*</u> Principals are scarce, so $U = \overline{U} = 7$ , but the efficient effort level does not depend on who is scarce, so it is still high. The agent is risk averse, so he is paid a flat wage. The wage satisfies the participation constraint $\sqrt{w} - 2.4 = 7$, if the method is risky. The contract specifies a wage of 88.4 (rounded) for the risky method and 0 for the safe. Profit is 48.6 $(= 0.5(49) + 0.5(225) - 88.4)$.

(d) If agents are the scarce factor, and principals compete for them, what will the contract be when the principal cannot observe effort? What will the agent's utility and the principal's profit be in this situation?

<u>*Answer.*</u> A boiling in oil contract can be used. Set either w(0) = -1000 or w(100) = -1000, which induces the agent to pick the risky method. In order to protect the agent from risk, the wage should be flat except for those outputs, so $w(49) = w(225) = 137$. $\pi = 0$, since agents are scarce. U= 9.3, from part (b).

**7.5. Worker Effort**

A worker can be $Careful$ or $Careless$, efforts which generate mistakes with probabilities 0.25 and 0.75. His utility function is $U = 100 - 10/w - x$, where $w$ is his wage and $x$ takes the value 2 if he is careful, and 0 otherwise. Whether a mistake is made is contractible, but effort is not. Risk-neutral employers compete for the worker, and his output is worth 0 if a mistake is made and 20 otherwise. No computation is needed for any part of this problem.

(a) Will the worker be paid anything if he makes a mistake?

<u>*Answer.*</u> Yes. He is risk averse, unlike the principal, so his wage should be even across states.

(b) Will the worker be paid more if he does not make a mistake?

<u>*Answer.*</u> Yes. Careful effort is efficient, and lack of mistakes is a good statistic for careful effort, which makes it useful for incentive compatibility.

(c) How would the contract be affected if employers were also risk averse?

<u>*Answer.*</u> The wage would vary more across states, because the workers should be less insured— and perhaps should even be insuring the employer.

(d) What would the contract look like if a third category, "slight mistake," with an output of 19, occurs with probability 0.1 after *Careless* effort and with probability zero after *Careful* effort?

<u>*Answer.*</u> The contract would pay equal amounts whether or not a mistake was made, but zero if a slight mistake was made, a "boiling in oil" contract.

**7.7. Optimal Compensation**

An agent's utility function is U =(log(wage) - effort). What should his compensation scheme be if different (output,effort) pairs have the probabilities in Table 8?

(a) The agent should be paid exactly his output.

@(b) The same wage should be paid for outputs of 1 and 100.

(c) The agent should receive more for an output of 100 than of 1, but should receive still lower pay if output is 2.

(d) None of the above.

**Table 8: Output Probabilities**

| | | **Output** | | |
|---|---|---|---|---|
| | | 1 | 2 | 100 |
| **Effort** | *High* | 0.5 | 0 | 0.5 |
| | *Low* | 0.1 | 0.8 | 0.1 |

**7.9. Hiring a Lawyer**

A one-man firm with concave utility function $U(X)$ hires a lawyer to sue a customer for breach of contract. The lawyer is risk-neutral and effort averse, with a convex disutility of effort. What can you say about the optimal contract? What would be the practical problem with such a contract, if it were legal?

Answer. The contract should give the firm a lump-sum payment and let the lawyer collect whatever he can from the lawsuit. The problem is that the firm would not have any incentive to help win the case.

**7.11. Constraints Again**

Suppose an agent has the utility function $U = log(w) - e$, where $e$ can take the levels 1 or 3, and a reservation utility of $\overline{U}$. The principal is risk-neutral. Denote the agent's wage conditioned on output as $\underline{w}$ if output is 0 and $\overline{w}$ if output is 100. Only the agent observes his effort. Principals compete for agents, and outputs occur according to Table 11.

**Table 11: Efforts and Outputs**

| | **Probability of Outputs** | |
|---|---|---|
| **Effort** | 0 | 100 |
| $Low(e = 1)$ | 0.9 | 0.1 |
| $High\ (e = 3)$ | 0.5 | 0.5 |

What conditions must the optimal contract satisfy, given that the principal can only observe output, not effort? You do not need to solve out for the optimal contract– just provide the equations which would have to be true. Do not just provide inequalities– if the condition is a binding constraint, state it as an equation.

Answer. This is a tricky question because it turns out with these numbers that low effort ($e = 1$) is optimal. In that case, the optimal contract is simple: a flat wage. Because principals compete, a zero- profit constraint must be satisfied, and $w = 0.9(0) + 0.1(100) = 10$. The incentive compatibility constraint is an inequality that is not binding: $U(e = 1) = log(10) - 1 \geq U(e = 3) = log(10) - 3$. The agent's utility is then

$$log(10) - 1 \approx 2.3 - 1 = 1.3.$$

For a high-effort contract, both a zero-profit and an incentive compatibility constraint must be binding. The zero profit constraint says

$$0.5(0) + 0.5(100) = 0.5\underline{w} + 0.5\overline{w},$$

so $\overline{w} = 100 - \underline{w}$.

The incentive compatibility constraint is

$$0.5log(\underline{w}) + 0.5log(\overline{w}) - 3 = 0.9log(\underline{w}) + 0.1log(\overline{w}) - 1.$$

That is the constraint, which must be an equality since principals are competing to offer the highest-utility contract to the agent (subject to the zero-profit constraint). Solving out a bit further, $-4log(\underline{w}) + 4log(\overline{w}) = 20$, so $log(\overline{w}/\underline{w}) = 5$, $\overline{w}/\underline{w} = Exp(5) \approx 148$ and $\overline{w} \approx 148\underline{w}$.

Equating our two equations for $\overline{w}$ yields

$$\overline{w} = 100 - \underline{w} \approx 148\underline{w}$$

so $\underline{w} \approx 100/149$. In turn, $\overline{w} \approx 100 - 100/149$.

What is the agent's utility from that? It is about

$$.5log(100/149) + .5log(100 - 100/149) - 3 \approx .5(-.4) + .5(4.6) - 3 = -0.9.$$

Thus, the principal gets zero profit with either high or low effort, but the agent gets lower utility from high effort.

**The First Best**

We could also work out the first best for this situation. For low effort, the agent's utility is the same as in the second-best, since in neither case does he bear risk, so $U(loweffort) \approx 1.3$. For high effort, in the first-best the agent gets a flat wage equal to the expected value of output, 50, so his utility is $U(higheffort) = log(50) - 3 \approx 3.91 - 3 = 0.91$. Thus, in the first-best, low effort is still better, but the utilities are closer than in the second-best.