Chapter 7 Moral Hazard: Hidden Actions

7.1 Categories of Asymmetric Information Models

♦ We will make heavy use of the principal-agent model.

- ○ The principal hires an agent to perform a task,

  and the agent acquires an informational advantage about his type,

  his actions, or the outside world at some point in the game.

- ○ It is usually assumed that the players can make a binding contract

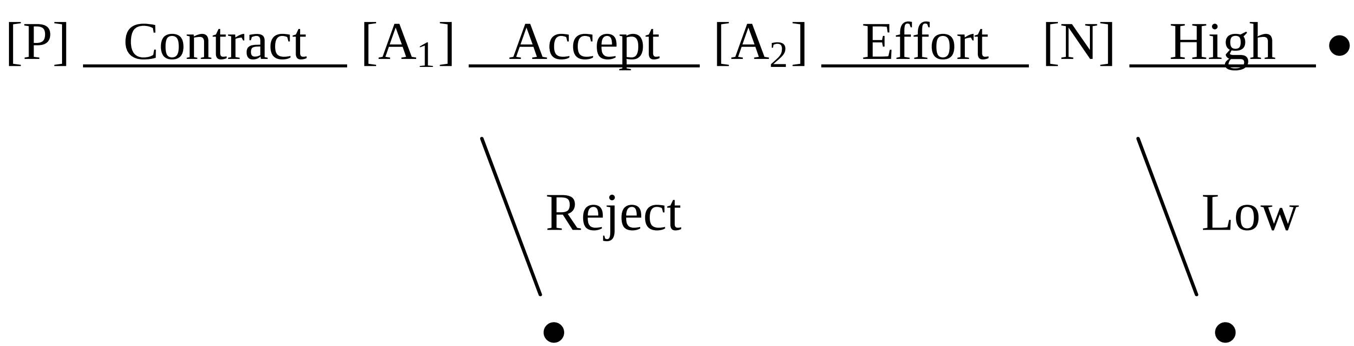
  at some point in the game.

- The <u>principal</u> (or uninformed player) is the player
  who has the <u>coarser</u> information partition.

- The <u>agent</u> (or informed player) is the player
  who has the <u>finer</u> information partition.

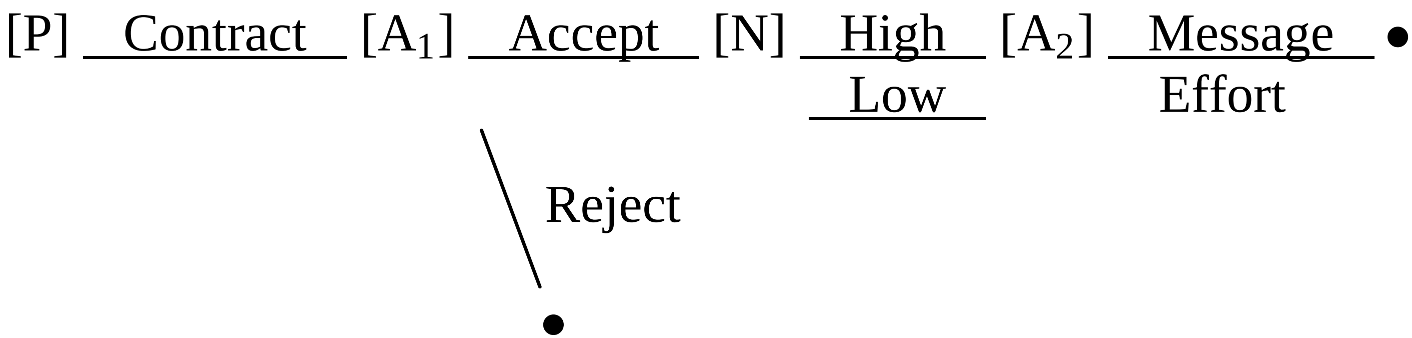
- Categories of asymmetric information models

  - Moral hazard with hidden actions

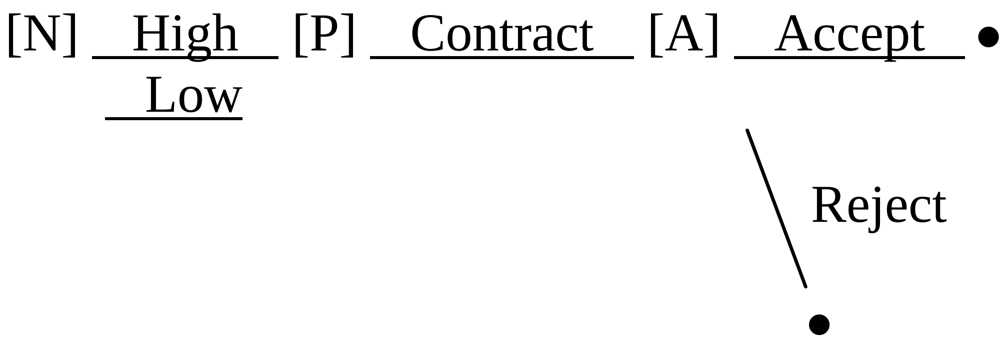


√ The moral hazard models are games of complete information with uncertainty.

- Postcontractual hidden knowledge



○ Adverse selection



√ Adverse selection models have incomplete information.

○ Signalling

[N] High [A_1] Signal [P] Contract [A_2] Accept ●

Low

Reject

●

√ A "signal" is different from a "message"

because it is not a costless statement, but a <u>costly action</u>.

- Screening

[N] High [P] Contract [A_1] Accept [A_2] Signal •

Low

Reject

•

√ If the worker acquires his credentials <u>in response to a wage offer</u> made by the employer, the problem is screening.

√ Many economists do not realize that screening and signalling are different and use the terms <u>interchangeably</u>.

7.2 A Principal-Agent Model: The Production Game

- The Production Game

  - Players

    - √ the principal and the agent

  - The order of play

    1. The principal <u>offers</u> the agent a wage w.
    2. The agent <u>decides</u> whether to accept or reject the contract.
    3. If the agent <u>accepts</u>, he exerts effort e.
    4. Output equals $q(e)$, where $q' > 0$.

- Payoffs

  - If the agent <u>rejects</u> the contract,

    then
$$\pi_{agent} = \bar{U} \quad \text{and} \quad \pi_{principal} = 0.$$

  - If the agent <u>accepts</u> the contract,

    then
$$\pi_{agent} = U(e, w) \quad \text{and} \quad \pi_{principal} = V(q - w),$$

    where $\partial U/\partial e < 0$, $\partial U/\partial w > 0$, and $V' > 0$.

- An assumption common to most principal-agent models

  - Other principals compete to employ the agent,

    so the principal's equilibrium profit equals zero.

  - Or many agents compete to work for the principal,

    so the agent's equilibrium utility equals the minimum

    for which he will accept the job, called the reservation utility, $\bar{U}$.

- Production Game I: Full Information
  - Every move is common knowledge and the contract is a function $w(e)$.
  - The principal must decide what he wants the agent to do and what incentive to give him to do it.
  - The agent must be paid some amount $\tilde{w}(e)$ to exert effort e, where $U(e, \tilde{w}(e)) = \bar{U}$.
  - The principal's problem is

$$\underset{e}{Maximize} \quad V(q(e) - \tilde{w}(e)).$$

- At the optimal effort level, e^*,

  the <u>marginal utility</u> to the agent which would result

  if he kept all the marginal output from extra effort

  equals the <u>marginal disutility</u> to him of that effort.

  √ $(\partial U/\partial \tilde{w})\,(\partial q/\partial e) \;=\; -\,\partial U/\partial e$

  √ $q(e)$ denotes the <u>monetary value</u> of output at an effort level e.

- Under <u>perfect competition</u> among the principals, the profits are <u>zero</u>.

  - √ at the profit-maximizing effort e^*

$$\tilde{w}(e^*) = q(e^*)$$

$$U(e^*, q(e^*)) = \bar{U}$$

  - √ The principal selects the point (e^*, w^*)

    on the indifference curve $\bar{U}$.

- The principal must then <u>design</u> a contract that will <u>induce</u> the agent

  to choose this effort level.

- The following contracts are equally <u>effective</u> under full information.

  - ✓ The <u>forcing contract</u> sets
$$w(e^*) = w^* \quad \text{and} \quad w(\mathrm{e} \neq e^*) = 0.$$

  - ✓ The <u>threshold contract</u> sets
$$w(e \geq e^*) = w^* \quad \text{and} \quad w(e < e^*) = 0.$$

  - ✓ The <u>linear contract</u> sets
$$w(e) = \alpha + \beta e,$$

    where α and β are chosen so that $w^* = \alpha + \beta e^*$ and

    the contract line is <u>tangent</u> to the indifference curve $\bar{U}$ at e^*.

- Utility function $U(e, w) = \log(w - e^2)$ is also a quasilinear function,

  because it is just a monotonic function of $U(e, w) = w - e^2$.

- Utility function $U(e, w) = \log(w - e^2)$ is concave in w,

  so it represents a risk-averse agent.

- As with utility function $U(e, w) = w - e^2$,

  the optimal effort does not depend on the agent's wealth w.

- ♦ Production Game II: Full Information

  - ○ Every move is <u>common knowledge</u> and the contract is a <u>function</u> $w(e)$.

  - ○ The agent moves first.

    - √ The <u>agent</u>, not the principal, proposes the contract.

  - ○ The order of play

    1 The agent <u>offers</u> the principal a contract $w(e)$.

    2 The principal <u>decides</u> whether to accept or reject the contract.

    3 If the principal <u>accepts</u>, the agent exerts effort e.

    4 Output equals $q(e)$, where $q' > 0$.

- In this game, the agent has all the <u>bargaining power</u>, not the principal.

  - √ The agent will maximize his own payoff

    by driving the principal to exactly <u>zero</u> profits.

  - √ $w(e) = q(e)$

- The maximization problem for the <u>agent</u> can be written as

$$\underset{e}{Maximize} \quad U(e, q(e)).$$

- The optimality equation is <u>identical</u> in Production Games I and II.

  - √ At the optimal effort level, e^*,

    the <u>marginal utility</u> of the money derived from marginal effort

    equals the <u>marginal disutility</u> of effort.

  - √ $(\partial U/\partial w)\,(\partial q/\partial e) \;=\; -\,\partial U/\partial e$

- Although the form of the optimality equation is the <u>same</u>,

  the <u>optimal effort</u> might not be, because <u>except</u> in the special case in which the agent's reservation payoff in Production Game I <u>equals</u> his equilibrium payoff in Production Game II,

  the agent ends up with <u>higher wealth</u> if he has all the bargaining power.

√ If the utility function is <u>not</u> quasilinear,

the <u>wealth effect</u> will change the optimal effort.

- If utility is <u>quasilinear</u>,

  the efficient effort level is <u>independent</u> of

  which side has the bargaining power

  because the gains from efficient production are <u>independent</u> of

  how those gains are <u>distributed</u>

  so long as each party has no incentive to <u>abandon</u> the relationship.

  √ This is the same lesson as the <u>Coase Theorem's</u>:

  under general conditions the activities undertaken will be <u>efficient</u>

  and <u>independent</u> of the distribution of <u>property rights</u>.

- ♦ Production Game III: A <u>Flat Wage</u> under Certainty

  - ○ The principal can condition the wage <u>neither</u> on effort <u>nor</u> on output.

    - √ The principal observes <u>neither</u> effort <u>nor</u> output,

      so information is asymmetric.

  - ○ The outcome of Production Game III is simple and <u>inefficient</u>.

    - √ If the wage is nonnegative,

      the agent <u>accepts</u> the job and exerts <u>zero</u> effort,

      so the principal offers a <u>wage of zero</u>.

- Moral hazard

  - √ the <u>problem</u> of the <u>agent</u> choosing the wrong action because the principal <u>cannot</u> use the contract to punish him

  - √ the <u>danger</u> to the <u>principal</u> that the agent, constrained only by his <u>morality</u>, not punishments, <u>cannot</u> be trusted to behave as he ought

  - √ a temptation for the <u>agent</u>, a <u>hazard</u> to his <u>morals</u>

- A clever contract can overcome moral hazard

  by conditioning the wage

  on something that is observable and correlated with effort,

  such as output.

- ♦ Production Game IV: An Output-based Wage under <u>Certainty</u>

  - ○ The principal <u>cannot</u> observe effort but <u>can</u> observe output and specify the <u>contract</u> to be $w(q)$.

  - ○ It is <u>possible</u> to achieve the efficient effort level e^* despite the unobservability of effort.

    - √ The principal starts by finding the optimal effort level e^*.

    - √ $q^* = q(e^*)$

- √ To give the agent the proper incentives,

  the contract must reward him when output is q^*.

- √ A variety of contracts could be used.

- √ The forcing contract, for example, would be any wage function such that

$$U(e^*, w(q^*)) = \bar{U} \quad \text{and} \quad U(e, w(q)) < \bar{U} \ \text{ for } \ e \neq e^*.$$

- ○ The unobservability of effort is not a problem in itself,

  if the contract can be conditioned on something

  which is observable and perfectly correlated with effort.

- Production Game V: Output-based Wage under Uncertainty

  - The principal cannot observe effort but can observe output and specify the contract to be $w(q)$.

  - Output, however, is a function $q(e, \theta)$ both of effort and the state of the world $\theta \in \mathbf{R}$, which is chosen by Nature according to the probability density $f(\theta)$.

  - The principal cannot deduce $e \neq e^*$ from $q \neq q^*$.

- Even if the contract does <u>induce</u> the agent to choose e^*,

  if it does so by penalizing him <u>heavily</u> when $q \neq q^*$,

  it will be <u>expensive</u> for the principal.

  √ The agent's expected utility must be kept equal to $\bar{U}$.

  √ If the agent is sometimes paid a <u>low</u> wage

  because output happens not to equal q^* despite his correct effort,

  he must be paid <u>more</u> when output does equal q^* to make up for it.

  √ There is a <u>tradeoff</u> between <u>incentives</u> and <u>insurance against risk</u>.

- Moral hazard is a <u>problem</u>

  when $q(e)$ is <u>not</u> a one-to-one function and

  a single value of e might result in any of a number of values of q,

  depending on the value of θ.

  √ The output function is <u>not</u> invertible.

- The combination of <u>unobservable effort</u> and <u>lack of invertibility</u> means

  that <u>no contract</u> can induce the agent to put forth

  the <u>efficient</u> effort level without incurring <u>extra costs</u>,

  which usually take the form of <u>extra risk</u> imposed on the agent.

- We will still try to find a <u>contract</u> that is <u>efficient</u>

  in the sense of maximizing welfare

  given the <u>informational constraints</u>.

- The terms "first-best" and "second-best" are used to distinguish these two kinds of optimality.

  - √ A <u>first-best contract</u> achieves the <u>same allocation</u> as the contract that is optimal when the principal and the agent have the <u>same</u> information set and all variables are <u>contractible</u>.

  - √ A <u>second-best contract</u> is Pareto optimal given information asymmetry and constraints on writing contracts.

  - √ The <u>difference</u> in welfare between the first-best and the second-best is the <u>cost of the agency problem</u>.

- How do we find a second-best contract?

  - √ Because of the tremendous variety of possible contracts,
    finding the optimal contract
    when a forcing contract cannot be used
    is a hard problem without general answers.

  - √ The rest of the chapter will show
    how the problem may be approached,
    if not actually solved.

7.3 The Incentive Compatibility and Participation Constraints

- ♦ The Participation Constraint and the Incentive Compatibility Contraint

  - ○ The principal's problem is

$$\underset{w(\cdot)}{Maximize} \quad EV(q(\tilde{e}, \theta) - w(q(\tilde{e}, \theta)))$$

subject to

$$\tilde{e} = \underset{e}{argmax} \quad EU(e, w(q(e, \theta)))$$

(incentive compatibility constraint)

$$EU(\tilde{e}, w(q(\tilde{e}, \theta))) \geq \bar{U}$$

(participation constraint).

  - √ the first-order condition approach

- The Three-Step Procedure

    - The first step is to find for <u>each</u> possible effort level the <u>set of wage contracts</u> that <u>induce</u> the agent to choose <u>that</u> effort level.

    - The second step is to find the <u>contract</u> which <u>supports</u> <u>that</u> effort level at the <u>lowest cost</u> to the principal.

    - The third step is to choose the <u>effort level</u> that <u>maximizes</u> profits, given the necessity to support <u>that</u> effort with the costly <u>wage contract</u> from the second step.

√ Mathematically, the problem of finding the <u>least cost</u> $C(\tilde{e})$ of <u>supporting</u> the effort level $\tilde{e}$ combines <u>steps one and two</u>.

$$C(\tilde{e}) = \underset{w(\,\cdot\,)}{Minimum} \qquad Ew(q(\tilde{e}, \theta))$$

subject to

$$\tilde{e} = \underset{e}{argmax} \quad EU(e, w(q(e, \theta)))$$

$$EU(\tilde{e}, w(q(\tilde{e}, \theta))) \geq \bar{U}$$

√ Step three takes the principal's problem of maximizing his payoff, and restates it as

$$\underset{\tilde{e}}{Maximize} \quad EV(q(\tilde{e}, \theta) - C(\tilde{e})). \qquad (7.27)$$

√ After finding which contract most cheaply induces each effort, the principal discovers the optimal effort by solving problem (7.27).

- ○ Breaking the problem into parts makes it easier to solve.

- ○ Perhaps the most important <u>lesson</u> of the three-step procedure is

  to reinforce the points

  - √ that the <u>goal</u> of the contract is to <u>induce</u> the agent

    to choose a particular effort level

    and

  - √ that asymmetric information increases the <u>cost</u> of the inducements.

7.4 Optimal Contracts: The Broadway Game

- A peculiarity of optimal contracts
  - Sometimes the agent's reward should <u>not</u> increase with his output.

- Broadway Game I
  - Players
    - √ producer and investors

○ The order of play

1 The investors offer a <u>wage contract</u> $w(q)$ as a function of <u>revenue</u> q.

2 The producer accepts or rejects the contract.

3 The producer chooses: *Embezzle* or *Do not embezzle*.

4 Nature picks the <u>state of the world</u> to be *Success* or *Failure* with <u>equal</u> probability.

√ Revenues (or profits)

State of the World

Effort

	Failure (0.5)	*Success* (0.5)
Embezzle	– 100	+ 100
Do not embezzle	– 100	+ 500

- Payoffs

  - The producer is <u>risk averse</u> and the investors are <u>risk neutral</u>.

  - The producer's payoff is $U(100)$ if he <u>rejects</u> the contract,

    where $U' > 0$ and $U'' < 0$,

    and the investors' payoff is 0.

  - Otherwise,

$$\pi_{producer} = \begin{cases} U(w(q)+50) & \text{if he embezzles} \\ U(w(q)) & \text{if he is honest} \end{cases}$$

$$\pi_{investors} = q - w(q)$$

- Boiling-in-oil contract

  - The investors will observe -100, $+100$, or $+500$.

    - $w(-100)$, $w(+100)$, and $w(+500)$

  - The producer's <u>expected payoffs</u>

    - $\pi(\textit{Do not embezzle}) = 0.5U(w(-100)) + 0.5U(w(+500))$

    - $\pi(\textit{Embezzle}) = 0.5\ U(w(-100) + 50) + 0.5\ U(w(+100) + 50)$

- The <u>incentive compatibility</u> constraint

  - √ π (*Do not embezzle*) $\geq$ π (*Embezzle*)

- The <u>participation</u> constraint

  - √ π (*Do not embezzle*) $\geq$ $U(100)$

- The investors want to impose as little risk on the producer as possible, since he requires a higher expected wage for higher risk.

  - √ $w(-100) = w(+500)$, which provides full insurance.

  - √ The outcome $+100$ cannot occur unless the producer chooses the undesirable action.

- The following boiling-in-oil contract provides both riskless wages and effective incentives.

  - √ $w(+500) = 100$

    $w(-100) = 100$

    $w(+100) = -\infty$

  - √ The participation constraint is satisfied, and is binding.

  - √ The incentive compatibility constraint is satisfied, and is nonbinding.

- The producer chooses *Do not embezzle* in equilibrium.

- The <u>cost</u> of the contract to the investors is 100 in equilibrium,

  so that their overall expected payoff is 100.

- ♦ The sufficient statistic condition

  - It says that for <u>incentive</u> purposes,

    if the agent's utility function is <u>separable</u> in effort and money,

    <u>wages</u> should be based on whatever evidence best indicates <u>effort</u>,

    and only incidentally on <u>output</u>.

  - In equilibrium, the datum $q = +500$ contains

    exactly the <u>same</u> information as the datum $q = -100$.

♦ Milder contracts

○ Two wages will be used in equilibrium,

a <u>low</u> wage $\underline{w}$ for an output of $q = +100$ and

a <u>high</u> wage $\overline{w}$ for any other output.

○ To find the <u>mildest</u> possible contract,

the modeller must specify a function for utility $U(w)$.

√ $U(w) = 100w - 0.1w^2$

- The <u>participation</u> constraint

  - √ Solving for the <u>full-insurance</u> high wage,

    we obtain

    $$\bar{w} \; = \; w(-100) \; = \; w(+500) \; = \; 100$$

    and a reservation utility of 9,000.

- The <u>incentive compatibility</u> constraint

  - √ Substituting into the incentive compatibility constraint,

    we obtain $\underline{w} \; \leq \; 5.6$.

  - √ A low wage of $-\infty$ is far more severe than what is needed.

- One of the oddities of Broadway Game I is

  that the wage is higher for an output of -100 than for an output

  of $+100$.

  - This illustrates the idea that the principal's aim is

    to reward input, not output.

  - If the principal pays more simply because output is higher,

    he is rewarding Nature, not the agent.

- Higher effort usually leads to higher output, but <u>not</u> always.

  Thus, higher pay is usually a good incentive, but <u>not</u> always,

  and sometimes <u>low</u> pay for <u>high</u> output actually punishes <u>slacking</u>.

- The decoupling of <u>reward</u> and <u>result</u> has broad applications.

- Shifting support scheme

  - The contract depends on the <u>support</u> of the <u>output distribution</u> being <u>different</u> when effort is <u>optimal</u> than when effort is other than optimal.

  - The set of possible <u>outcomes</u> under <u>optimal</u> effort must be <u>different</u> from the set of possible outcomes under any <u>other</u> effort level.

    - √ As a result, particular <u>outputs</u> show without doubt that the producer embezzled.

    - √ Very heavy <u>punishments</u> inflicted only for those outputs achieve the <u>first-best</u>.

- The <u>conditions</u> favoring boiling-in-oil contracts are
    - The agent is <u>not</u> very risk averse.
    - There are <u>outcomes</u> with <u>high</u> probability under <u>shirking</u> that have <u>low</u> probability under <u>optimal</u> effort.
    - The agent <u>can</u> be severely punished.
    - It is <u>credible</u> that the principal will <u>carry out</u> the severe punishment.

- Selling the Store
  - Another first-best contract that can sometimes be used is selling the store.
  - Under this arrangement, the agent buys the entire output for a flat fee paid to the principal, becoming the residual claimant.
  - This is equivalent to fully insuring the principal, since his payoff becomes independent of the moves of the agent and of Nature.

- The <u>drawbacks</u> are

  - √ that the producer might <u>not</u> be able to afford to pay the investors the flat price of 100,

    and

  - √ the producer might be <u>risk-averse</u> and incur a <u>heavy</u> utility cost in bearing the <u>entire risk</u>.

♦ Public Information That Hurts the Principal and the Agent

○ Having <u>more</u> public information available can <u>hurt</u> both players.

○ Revenues (or profits) in Broadway Game II

State of the World

Effort

	Failure (0.5)	*Minor Success* (0.3)	*Major Success* (0.2)
Embezzle	– 100	– 100	+ 400
Do not embezzle	– 100	+ 450	+ 575

√ Each player's initial <u>information partition</u> is

({*Failure*, *Minor Success*, *Major Success*}).

○ Under the <u>optimal</u> contract,

$$w(-100) = w(+450) = w(+575) > w(+400) + 50.$$

- √ This is so because the producer is <u>risk-averse</u> and only the datum $q = +400$ is <u>proof</u> that the producer embezzled.

- √ The <u>optimal</u> contract must do <u>two</u> things: <u>deter</u> embezzlement and <u>pay</u> the producer as predictable a wage as possible.

- √ $w(-100) = w(+450) = w(+575) = 100$

  $w(+400) = -\infty$

- √ The punishment would <u>not</u> have to be infinitely severe, and the <u>minimum effective punishment</u> could be calculated.

- √ The producer chooses *Do not embezzle* in equilibrium.

- √ The investors' expected payoff is 100 in equilibrium.

- Broadway Game III

  - √ Before the agent takes his action, both he and the principal can tell whether the show will be a major success or not.

  - √ Each player's initial information partition is ({*Failure*, *Minor Success*}, {*Major Success*}).

√ If the investors could still hire the producer and prevent him from embezzling at a cost of 100, the payoff from investing in a major success would be 475.

But the payoff from investing in a show given the information set {*Failure*, *Minor Success*} would be about 6.25, which is still positive.

So the improvement in information is no help with respect to the decision of when to invest.

√ The <u>refinement</u> does, however, <u>ruin</u> the producer's <u>incentives</u>.

If he observes {*Failure*, *Minor Success*},

he is free to embezzle without fear of the oil-boiling output

of $+400$.

√ <u>Better</u> information <u>reduces</u> welfare,

because it <u>increases</u> the producer's <u>temptation</u> to misbehave.