## 8.4 Renegotiation: The Repossession Game

- The players have signed a binding contract,

  but in a subsequent subgame,

  both might agree to scrap the old contract and write a new one,

  using the old contract as a starting point in their negotiations.

- Here we use a model of hidden actions to illustrate renegotiation,

  a model in which a bank that wants to lend money to a consumer

  to buy a car must worry about whether he will work hard

  enough to repay the loan.

- As we will see, the outcome is Pareto superior

  if renegotiation is not possible.

- ♦ Repossession Game I
  - Players
    - ✓ a bank and a consumer

- The order of play

1. The bank can do nothing or it can at cost 11 offer the consumer an auto loan which allows him to buy a car that costs 11, but requires him to pay back $L$ or lose possession of the car to the bank.

2. The consumer accepts the loan and buys the car, or rejects it.

3. The consumer chooses to *Work*, for an income of 15, or *Play*, for an income of 8. The disutility of work is 5.

4. The consumer repays the loan or defaults.

5. If the bank has not been paid $L$, it repossesses the car.

- Payoffs

  - √ If the consumer chooses *Work*,

    his income is $W = 15$ and his disutility of effort is $D = 5$.

  - √ If the consumer chooses *Play*, then $W = 8$ and $D = 0$.

  - √ If the bank does not make any loan or the consumer rejects it,

    the bank's payoff is zero and the consumer's payoff is $W - D$.

  - √ The value of the car is 12 to the consumer and 7 to the bank,

    so the bank's payoff if the loan is made is

$$\pi_{bank} = \begin{cases} L - 11 & \text{if the loan is repaid} \\ 7 - 11 & \text{if the car is repossessed.} \end{cases}$$

√ The consumer's payoff is

$$\pi_{consumer} = \begin{cases} W + 12 - L - D & \text{if the loan is repaid} \\ W - D & \text{if the car is repossessed.} \end{cases}$$

- The model allows <u>commitment</u> in the sense of

  legally binding agreements over transfers of money and wealth

  but it does <u>not</u> allow the consumer to commit <u>directly</u> to *Work*.

- It does <u>not</u> allow <u>renegotiation</u>.

- In equilibrium
  - The bank's <u>strategy</u> is to offer $L = 12$.
  - The consumer's <u>strategy</u>
    - √ *Accept* if $L \leq 12$
    - √ *Work* if $L \leq 12$ and he has accepted the loan or

      if he has rejected the loan

      (or if the bank does not make any loan)
    - √ *Repay* if $W + 12 - L - D \geq W - D$

- The equilibrium outcome is that the bank offers $L = 12$,

  the concumer accepts, he works, and he repays the loan.

- The bank's equilibrium payoff is $1$.

- This outcome is efficient because the consumer does buy the car,

  which he values at more than its cost to the car dealer.

- The bank ends up with the surplus,

  because of our assumption that the bank has

  all the bargaining power over the terms of the loan.

- Repossession Game II

  - Players

    - √ a bank and a consumer

  - The order of play

    1 The bank can do nothing or

    it can at cost 11 offer the consumer an <u>auto loan</u> which allows him to buy a car that costs 11, but requires him to pay back $L$ or lose possession of the car to the bank.

    2 The consumer accepts the loan and buys the car, or rejects it.

3 The consumer chooses to *Work*, for an income of 15, or *Play*, for an income of 8.

The disutility of work is 5.

4 The consumer repays the loan or defaults.

4a The bank offers to settle for an amount *S* and leave possession of the car to the consumer.

4b The consumer accepts or rejects the <u>settlement</u> *S*.

5 If the bank has not been paid *L* or *S*, it repossesses the car.

- Payoffs

  - √ If the consumer chooses *Work*,
    his income is $W = 15$ and his disutility of effort is $D = 5$.

  - √ If the consumer chooses *Play*, then $W = 8$ and $D = 0$.

  - √ If the bank does not make any loan or the consumer rejects it,
    the bank's payoff is zero and the consumer's payoff is $W - D$.

- ✓ The value of the car is 12 to the consumer and 7 to the bank, so the bank's payoff if the loan is made is

$$\pi_{bank} = \begin{cases} L - 11 & \text{if the original loan is repaid} \\ S - 11 & \text{if a settlement is made} \\ 7 - 11 & \text{if the car is repossessed.} \end{cases}$$

- ✓ The consumer's payoff is

$$\pi_{consumer} = \begin{cases} W + 12 - L - D & \text{if the original loan is repaid} \\ W + 12 - S - D & \text{if a settlement is made} \\ W - D & \text{if the car is repossessed.} \end{cases}$$

- ○ The model does allow renegotiation.

- In equilibrium

  - The equilibrium in Repossession Game I breaks down in Repossession Game II.

    - √ The consumer would deviate by choosing *Play*.

    - √ The bank chooses to renegotiate and offer $S = 8$.

    - √ The offer is accepted by the consumer.

    - √ Looking ahead to this, the bank refuses to make the loan.

- The bank's <u>strategy</u> in equilibrium

  - √ It does <u>not</u> offer a loan at all.

  - √ If it did offer a loan and the consumer accepted and defaulted, then it offers

    $S = 12$ if the consumer chose *Work*

    and

    $S = 8$ if the consumer chose *Play*.

- The consumer's strategy in equilibrium

  - √ *Accept* any loan made, whatever the value of $L$

  - √ *Work* if he rejected the loan

    (or if the bank does not make any loan)

    *Play and Default* otherwise

  - √ *Accept* a settlement offer of

    $S = 12$ if he chose *Work*

    and

    $S = 8$ if he chose *Play*

- The equilibrium outcome is that the bank does not offer a loan and the consumer chooses *Work*.

- Renegotiation turns out to be harmful, because it results in an equilibrium in which the bank refuses to make the loan, reducing the payoffs of the bank and the consumer to (0,10) instead of (1,10).

  √ The gains from trade vanish.

- Renegotiation is paradoxical.

  - In the subgame starting with consumer default,

    it <u>increases</u> efficiency,

    by allowing the players to make a <u>Pareto improvement</u>

    over an inefficient punishment.

  - In the game as a whole, however, it <u>reduces</u> efficiency

    by preventing players from using punishments

    to deter inefficient actions.

- The Repossession Game illustrates other ideas too.

  - It is a game of perfect information,

    but it has the feel of a game of moral hazard with hidden actions.

  - This is because it has an implicit bankruptcy constraint,

    so that the contract cannot sufficiently punish the consumer

    for an inefficient choice of effort.

  - Restricting the strategy space has the same effect

    as restricting the information available to a player.

  - It is another example of the distinction between

    observability and contractibility.

## 8.5 State-Space Diagrams: Insurance Games I and II

- Suppose Smith (the agent) is considering buying theft insurance for a car with a value of 12.

- A state-space diagram
  - A diagram whose axes measure the values of one variable in two different states of the world
  - His endowment is $\omega = (12, 0)$.

♦ Insurance Game I: Observable Care

○ Players

√ Smith and two insurance companies

○ The order of play

1 Smith chooses to be either *Careful* or *Careless*, observed by the insurance company.

2 Insurance company 1 offers a contract $(x, y)$, in which Smith pays premium $x$ and receives compensation $y$ if there is a theft.

3 Insurance company 2 also offers a contract of the form $(x, y)$.

4 Smith picks a contract.

5 Nature chooses whether there is a theft,

with probability 0.5 if Smith is *Careful* or

0.75 if Smith is *Careless*.

- Payoffs

  - Smith is <u>risk-averse</u> and the insurance companies are <u>risk-neutral</u>.

  - The insurance company not picked by Smith has a payoff of zero.

  - Smith's utility function $U$ is such that $U' > 0$ and $U'' < 0$.

  - If Smith chooses *Careful*, the payoffs are

$$\pi_{Smith} = 0.5\,U(12 - x) + 0.5\,U(0 + y - x)$$

and

$$\pi_{company} = 0.5\,x + 0.5\,(x - y) \quad \text{for his insurer.}$$

- √ If Smith chooses *Careless*, the payoffs are

$$\pi_{Smith} \;=\; 0.25\,U(12-x) \;+\; 0.75\,U(0+y-x) \;+\; \epsilon$$

and

$$\pi_{company} \;=\; 0.25\,x \;+\; 0.75\,(x-y) \qquad \text{for his insurer.}$$

- The optimal contract with only the *Careful* type

  - If the insurance company can require Smith to park carefully,

    it offers him insurance at a premium of 6,

    with a payout of 12 if theft occurs,

    leaving him with an allocation of $C_1 = (6, 6)$.

    - √ $(x, y) = (6, 12)$

- This satisfies the <u>competition constraint</u>

  because it is the most attractive contract any company can offer

  without making losses.

  - √ An insurance policy $(x, y)$ is <u>actuarially fair</u>

    if the cost of the policy is precisely its expected value.

  - √ $x = 0.5y$

- Smith is <u>fully insured</u>.

  - √ His allocation is 6 no matter what happens.

- In equilibrium
  - Smith chooses to be *Careful*

    because he foresees that otherwise his insurance will be

    more expensive.
  - Edgeworth box
  - The company is risk-neutral,

    so its indifference curves are straight lines with a slope of $-1$.
  - Smith is risk-averse,

    so (if he is *Careful*) his indifference curves are closest to the origin

    on the $45^{o}$ line, where his wealth in the two states is equal.

√ the <u>slope</u> of an indifference curve

$$p_1 u(x_1) + p_2 u(x_2) = k$$

$$p_1 u'(x_1) dx_1 + p_2 u'(x_2) dx_2 = dk = 0$$

$$dx_2/dx_1 = -p_1 u'(x_1)/p_2 u'(x_2)$$

- The equilibrium contract is $C_1$.
  - √ It satisfies the competition constraint by generating the highest expected utility for Smith.
  - √ It allows nonnegative profits to the company.

♦ Insurance Game I is a game of symmetric information.

♦ Suppose that Smith's action is a noncontractible variable.

- We model the situation by putting Smith's move second.

- Insurance Game II: Unobservable Care

  - Players

    - √ Smith and two insurance companies

  - The order of play

    1 Insurance company 1 offers a contract of form ($x$, $y$), under which Smith pays premium $x$ and receives compensation $y$ if there is a theft.

    2 Insurance company 2 offers a contract of form ($x$, $y$).

3 Smith picks a contract.

4 Smith chooses either *Careful* or *Careless*.

5 Nature chooses whether there is a theft,

with probability 0.5 if Smith is *Careful* or

0.75 if Smith is *Careless*.

- Payoffs

  - √ Smith is risk-averse and the insurance companies are risk-neutral.

  - √ The insurance company not picked by Smith has a payoff of zero.

  - √ Smith's utility function $U$ is such that $U' > 0$ and $U'' < 0$.

  - √ If Smith chooses *Careful*, the payoffs are

$$\pi_{Smith} = 0.5\, U(12 - x) + 0.5\, U(0 + y - x)$$

and

$$\pi_{company} = 0.5\, x + 0.5\, (x - y) \quad \text{for his insurer.}$$

- √ If Smith chooses *Careless*, the payoffs are

$$\pi_{Smith} = 0.25\, U(12 - x) + 0.75\, U(0 + y - x) + \epsilon$$

and

$$\pi_{company} = 0.25\, x + 0.75\, (x - y) \quad \text{for his insurer.}$$

- No full-insurance contract will be offered.

  - If Smith is fully insured, his dominant strategy is *Careless*.

  - The company knows the probability of a theft is 0.75.

  - The insurance company must offer a contract with a premium of 9
    and a payout of 12 to prevent losses,
    which leaves Smith with an allocation $C_2 = (3, 3)$.

  - The insurance company's isoprofit curve swivels around $\omega$
    because that is the point at which the company's profit is
    independent of how probable it is that Smith's car will be stolen.

    - √ At point $\omega$, the company is not insuring him at all.

- Smith's indifference curve swivels around the intersection of

  the $\pi_s = 66$ curve with the $45^{o}$ line,

  because on that line the probability of theft does not affect

  his payoff.

- Smith would like to commit himself to being careful,

  but he cannot make his commitment credible.

- The outlook is bright because Smith chooses *Careful*

  if he only has partial insurance,

  as with contract $C_3$.

  - The moral hazard is "small"

    in the sense that Smith barely prefers *Careless*.

  - Deductibles and coinsurance

  - The solution of full insurance is "almost" reached.

- Even when the ideal of full insurance and efficient effort cannot be reached, there exists some best choice like $C_5$ in the set of feasible contracts, a second-best insurance contract that recognizes the constraints of informational asymmetry.