8.6 Joint Production by Many Agents: The Holmstrom Teams Model

- The existence of a group of agents results in destroying the effectiveness of the individual risk-sharing contracts, because observed output is a joint function of the unobserved effort of many agents.

- The actions of a group of players produce a joint output, and each player wishes that the others would carry out the costly actions.

- A team is a group of agents who independently choose effort levels that result in a single output for the entire group.

- Teams
  - Players
    - √ a principal and n agents
  - The order of play
    1. The principal offers a <u>contract</u> to each agent i of the form $w_i(q)$, where q is total output.
    2. The agents decide whether or not to accept the contract.
    3. The agents simultaneously pick effort levels e_i, $(i = 1, \ldots, n)$.
    4. Output is $q(e_1, \ldots, e_n)$.

- Payoffs
  - √ If any agent rejects the contract, all payoffs equal zero.
  - √ Otherwise,

$$\pi_{principal} = q - \sum_{i=1}^{n} w_i$$

and

$$\pi_i = w_i - v_i(e_i), \text{ where } v_i' > 0 \text{ and } v_i'' > 0.$$

- The principal can <u>observe</u> output.
- The team's problem is <u>cooperation</u> between agents.

- Efficient contracts

  - Denote the efficient vector of actions by e^*.

  - An efficient contract is

$$w_i(q) = \begin{cases} b_i & \text{if } q \geq q(e^*) \\ 0 & \text{if } q < q(e^*), \end{cases} \tag{8.9}$$

where $\sum_{i=1}^{n} b_i = q(e^*)$ and $b_i > v_i(e_i^*)$.

  - The teams model gives one reason to have a principal:

    he is the residual claimant who keeps the forfeited output.

- Budget balancing and Proposition 8.1

  - The budget-balancing constraint

    - √ The sum of the wages exactly equal the output.

  - If there is a budget-balancing constraint,

    no differentiable wage contract $w_i(q)$ generates

    an efficient Nash equilibrium.

    - √ Agent i's problem is

$$\underset{e_i}{Maximize} \quad w_i(q(e)) - v_i(e_i).$$

His first-order condition is

$$(dw_i/dq)\,(\partial q/\partial e_i) - dv_i/de_i = 0.$$

√ The Pareto optimum solves

$$\underset{e_1,\ldots,e_n}{Maximize} \qquad q(e) - \sum_{i=1}^{n} v_i(e_i).$$

The first-order condition is that the marginal dollar contribution equal the marginal disutility of effort:

$$\partial q/\partial e_i - dv_i/de_i = 0.$$

√ $dw_i/dq \neq 1$

Under budget balancing,

<u>not every agent</u> can receive the <u>entire</u> marginal increase in output.

- √ Because each agent bears the <u>entire burden</u> of his marginal effort and only <u>part</u> of the benefit, the contract <u>does not</u> achieve the first-best.

♦ Without budget balancing,

  if the agent shirked a little

  he would gain the entire leisure benefit from shirking,

  but he would lose his entire wage under the optimal contract

  in equation (8.9).

- With budget balancing and a linear utility function,

  the <u>Pareto optimum</u> maximizes the <u>sum</u> of utilities.

  - A Pareto efficient allocation is one where consumer 1 is as well-off

    as possible <u>given</u> consumer 2's level of utility.

    - ✓ Fix the utility of consumer 2 at $\bar{u}_2$.

  - $$\underset{e_1,\, e_2}{\textit{Maximize}} \quad w_1(q(e)) - v_1(e_1)$$

    subject to

    $$w_2(q(e)) - v_2(e_2) \geq \bar{u}_2$$

    and

    $$w_1(q(e)) + w_2(q(e)) = q(e)$$

- $$\underset{e_1,\, e_2}{\mathit{Maximize}} \qquad w_1(q(e)) \ - \ v_1(e_1)$$

  subject to

  $$q(e) \ - \ v_2(e_2) \ - \ \overline{u}_2 \ = \ w_1(q(e))$$

- $$\underset{e_1,\, e_2}{\mathit{Maximize}} \qquad q(e) \ - \ (v_1(e_1) + v_2(e_2)) \ - \ \overline{u}_2$$

- Discontinuities in Public Good Payoffs

  - There is a <u>free rider problem</u>

    if <u>several players</u> each pick a level of effort which increases

    the level of some <u>public good</u> whose benefits they share.

    - √ Noncooperatively, they choose effort levels

      <u>lower</u> than if they could make <u>binding promises</u>.

- Consider a situation in which n identical risk-neutral players produce a public good by expending their effort.

  - √ Let e_i represent player i's effort level, and let $q(e_1, \ldots, e_n)$ the amount of the public good produced, where q is a continuous function.

  - √ Player i's problem is

$$\underset{e_i}{Maximize} \quad q(e_1, \ldots, e_n) - e_i.$$

His first-order condition is

$$\partial q / \partial e_i - 1 = 0.$$

- √ The <u>greater</u>, first-best n-tuple vector of effort levels e^* is characterized by

$$\sum_{i=1}^{n}(\partial q/\partial e_i) - 1 = 0.$$

- ○ If the function q were <u>discontinuous</u> at e^* (for example, $q = 0$ if $e_i < e_i^*$ and $q = e_i$ if $e_i \geq e_i^*$ for any i), the strategy profile e^* could be a <u>Nash equilibrium</u>.

- ○ The <u>first-best</u> can be achieved because the <u>discontinuity</u> at e^* makes every player the marginal, decisive player.

- √ If he shirks a little, output falls drastically and with certainty.

- Either of the following two modifications restores the free rider problem and induces shirking:

  - √ Let q be a function not only of effort but of random noise. Nature moves after the players. Uncertainty makes the expected output a continuous function of effort.

  - √ Let players have incomplete information about the critical value. Nature moves before the players and chooses e^*. Incomplete information makes the estimated output a continuous function of effort.

- The <u>discontinuity</u> phenomenon is common.

  Examples include:

  - Effort in teams
    (Holmstrom [1982], Rasmusen [1987])

  - Entry deterrence by an oligopoly
    (Bernheim [1984b], Waldman [1987])

  - Output in oligopolies with trigger strategies
    (Porter [1983a])

  - Patent races

  - Tendering shares in a takeover
    (Grossman & Hart [1980])

  - Preferences for levels of a public good.

- Pareto optimum

  - $$\underset{e_1,\, e_2}{Maximize} \qquad q(e_1, e_2) - e_1$$

    subject to

    $$q(e_1, e_2) - e_2 = \bar{u}_2$$

  - To solve the maximization problem,

    we set up the Lagrangian function:

    $$L = q(e_1, e_2) - e_1 - \lambda\, \{q(e_1, e_2) - e_2 - \bar{u}_2\}.$$

We have the following set of simultaneous equations:

$$\partial L/\partial \lambda \;=\; -\{q(e_1, e_2) - e_2 - \overline{u}_2\} \;=\; 0$$

$$\partial L/\partial e_1 \;=\; \partial q/\partial e_1 \;-\; 1 \;-\; \lambda \partial q/\partial e_1 \;=\; 0 \tag{A1}$$

$$\partial L/\partial e_2 \;=\; \partial q/\partial e_2 \;-\; \lambda(\partial q/\partial e_2 - 1) \;=\; 0. \tag{A2}$$

Using expressions (A1) and (A2), we obtain

$$(1 \;-\; \lambda) \sum_{i=1}^{2} (\partial q/\partial e_i) \;=\; 1 \;-\; \lambda,$$

which leads to

$$\sum_{i=1}^{2} (\partial q/\partial e_i) - 1 \;=\; 0.$$

8.7 The Multitask Agency Problem

- Holmstrom and Milgrom (1991)
  - Often the principal wants the agent to split his time among several tasks, each with a separate output, rather than just working on one of them.
  - If the principal uses one of the incentive contracts to incentivize just one of the tasks, this "high-powered incentive" can result in the agent completely neglecting his other tasks and leave the principal worse off than under a flat wage.

- ♦ Multitasking I: Two Tasks, No Leisure

  - ○ Players

    - √ a principal and an agent

  - ○ The order of play

    1. The principal offers the agent

       either an <u>incentive contract</u> of the form $w(q_1)$ or

       a <u>monitoring contract</u> that pays m under which he pays the agent

       m_1 if he observes the agent working on Task 1 and

       m_2 if he observes the agent working on Task 2.

2 The agent decides whether or not to accept the contract.

3 The agent picks effort levels e_1 and e_2 for the two tasks

such that $\boldsymbol{e_1 + e_2 = 1}$,

where 1 denotes the total time available.

4 Outputs are $q_1(e_1)$ and $q_2(e_2)$,

where $dq_1/de_1 > 0$ and $dq_2/de_2 > 0$

but we <u>do not</u> require decreasing returns to effort.

- Payoffs

  - √ If the agent rejects the contract, all payoffs equal zero.

  - √ Otherwise,

    $$\pi_{principal} = q_1 + \beta q_2 - m - w - C$$

    and

    $$\pi_{agent} = m + w - e_1^2 - e_2^2,$$

    where C, the cost of monitoring, is $\bar{C}$ if a monitoring contract is used and zero otherwise.

  - √ β is a measure of the relative value of Task 2.

- The principal can <u>observe</u> the output from one of the agent's tasks (q_1) but <u>not</u> from the other (q_2).

- The <u>first best</u> can be found by choosing e_1 and e_2 (subject to $e_1 + e_2 = 1$) and C to <u>maximize</u> the <u>sum</u> of the payoffs.

  - $$\underset{e_1, e_2, C}{Maximize} \quad \pi_{principal} = q_1(e_1) + \beta q_2(e_2) - m - w - C$$

    subject to

    $$\pi_{agent} = m + w - e_1^2 - e_2^2 \geq \bar{U} = 0$$

    and

    $$e_1 + e_2 = 1$$

  - $$\underset{e_1, e_2, C}{Maximize} \quad \pi_{principal} + \pi_{agent} - \bar{U}$$

    subject to

    $$e_1 + e_2 = 1$$

- The first-best levels of the variables

  - $C^* = 0$

  - $$e_1^* = 0.5 + 0.25\,\{dq_1/de_1 - \beta(dq_2/de_2)\} \tag{8.19}$$

  - $e_2^* = 0.5 - 0.25\,\{dq_1/de_1 - \beta(dq_2/de_2)\}$

  - $q_i^* \equiv q_i(e_i^*)$

  - Define the minimum wage payment that would induce the agent to accept a contract requiring the first-best effort levels as

    $$w^* \equiv (e_1^*)^2 + (e_2^*)^2.$$

- Can an <u>incentive contract</u> achieve the first best?

  - A profit-maximizing <u>flat-wage</u> contract

    - √ $w(q_1) = w^o$ or the monitoring contract $\{w^o, w^o\}$

    - √ The agent chooses $e_1^o = e_2^o = 0.5$.

    - √ $w^o = 0.5$ satisfies the participation constraint.

- A sharing-rule incentive contract

  - √ $dw/dq_1 > 0$

  - √ The greater the agent's effort on Task 1, the less will be his effort on Task 2.

  - √ Even if extra effort on Task 1 could be achieved for free, the principal might not want it – and, in fact, he might be willing to pay to stop it.

- The simplest sharing-rule (incentive) contract

  - √ the linear contract

$$w(q_1) = a + bq_1$$

  - √ The agent will pick e_1 and e_2 to maximize

$$\pi_{agent} = a + bq_1(e_1) - e_1^2 - e_2^2$$

subject to $e_1 + e_2 = 1$.

  - √ $e_1^o = 0.5 + 0.25b(dq_1/de_1)$ (8.23)

√ If $e_1^* \geq 0.5$, the linear contract will <u>work</u> just fine.

The contract parameters a and b can be chosen

so that the linear-contract effort level in equation (8.23) is

the same as the <u>first-best</u> effort level in equation (8.19),

with a taking a value to extract all the surplus

so the participation constraint is barely satisfied.

√ If $e_1^* < 0.5$, the linear contract <u>cannot</u> achieve the first best

with a positive value for b.

The contract must actually <u>punish</u> the agent for high output!

- In equilibrium,

  the principal chooses some <u>contract</u> that elicits the <u>first-best</u> effort e^*, such as the forcing contract,

$$w(q_1 = q_1^*) = w^*$$

  and

$$w(q_1 = q_1^*) = 0.$$

- A monitoring contract
  - The cost $\bar{C}$ of monitoring is incurred.
  - The agent will pick e_1 and e_2 to maximize

$$\pi_{agent} = e_1 m_1 + e_2 m_2 - e_1^2 - e_2^2$$

subject to $e_1 + e_2 = 1$.

    - √ The principal finds the agent working on Task i with probability e_i.
    - √ $\pi_{agent} = e_1 m_1 + (1 - e_1) m_2 - e_1^2 - (1 - e_1)^2$
    - √ $d\pi_{agent}/de_1 = m_1 - m_2 - 2e_1 + 2(1 - e_1) = 0$

- If the principal wants the agent to pick e_1^*,

  he should choose m_1^* and m_2^* so that

$$m_1^* = 4e_1^* + m_2^* - 2.$$

  - √ the binding participation constraint

$$e_1^* m_1^* + (1 - e_1^*) m_2^* - (e_1^*)^2 - (1 - e_1^*)^2 = 0$$

- $m_1^* = 4e_1^* - 2(e_1^*)^2 - 1$

  $m_2^* = 1 - 2(e_1^*)^2$

  - √ $e_1^* > e_2^* \quad \Rightarrow \quad m_1^* > m_2^*$

  - √ $dm_1^*/de_1^* > 0$

  - √ $dm_2^*/de_1^* < 0$

♦ Multitasking II: Two Tasks Plus Leisure

- Players

  √ a principal and an agent

- The order of play

1 The principal offers the agent either an <u>incentive contract</u> of the form $w(q_1)$ or

a <u>monitoring contract</u> that pays m under which he pays the agent a base wage of $\bar{m}$ plus

m_1 if he observes the agent working on Task 1 and

m_2 if he observes the agent working on Task 2.

2 The agent decides whether or not to accept the contract.

3 The agent picks effort levels e_1 and e_2 for the two tasks.

4 Outputs are $q_1(e_1)$ and $q_2(e_2)$,

where $dq_1/de_1 > 0$ and $dq_2/de_2 > 0$

but we do not require decreasing returns to effort.

- Payoffs
  - If the agent rejects the contract, all payoffs equal zero.
  - Otherwise,

$$\pi_{principal} = q_1 + \beta q_2 - m - w - C$$

and

$$\pi_{agent} = m + w - e_1^2 - e_2^2,$$

where C, the cost of monitoring, is $\bar{C}$ if a monitoring contract is used and zero otherwise.

  - β is a measure of the relative value of Task 2.

- The principal can <u>observe</u> the output from one of the agent's tasks (q_1) but <u>not</u> from the other (q_2).

- $\boldsymbol{e_1 + e_2 \leq 1}$

    - √ The amount $(1 - e_1 - e_2)$ represents leisure,

      whose value we set equal to zero in the agent's utility function.

    - √ Here leisure represents not time off the job,

      but time on the job spent shirking rather than working.

- The first-best can be found by choosing e_1, e_2, and C to maximize the sum of the payoffs.

  - $$\underset{e_1, e_2, C}{\text{Maximize}} \quad \pi_{principal} = q_1(e_1) + \beta q_2(e_2) - m - w - C$$

    subject to

    $$\pi_{agent} = m + w - e_1^2 - e_2^2 \geq 0$$

    and

    $$e_1 + e_2 \leq 1$$

  - $$\underset{e_1, e_2, C}{\text{Maximize}} \quad q_1(e_1) + \beta q_2(e_2) - C - e_1^2 - e_2^2$$

    subject to

    $$e_1 + e_2 \leq 1$$

- The first-best levels of the variables

  - √ $C^{**} = 0$
  - √ $e_1^{**} = ?$
  - √ $e_2^{**} = ?$
  - √ $q_i^{**} \equiv q_i(e_i^{**})$
  - √ Define the minimum wage payment that would induce the agent to accept a contract requiring the <u>first-best</u> effort levels as

    $w^{**} \equiv (e_1^{**})^2 + (e_2^{**})^2.$

  - √ <u>Positive</u> leisure for the agent in the first-best is a <u>realistic</u> case.

- Can an incentive contract achieve the first best?
  - A flat-wage contract
    - √ $w(q_1) = w^{oo}$ or the monitoring contract $\{w^{oo}, w^{oo}\}$
    - √ The agent chooses $e_1^{oo} = e_2^{oo} = 0$.
    - √ A low-powered incentive contract is disastrous, because pulling the agent away from high effort on Task I does not leave him working harder on Task 2.

- A high-powered sharing-rule incentive contract

  - √ $dw/dq_1 > 0$

  - √ The first-best is unreachable since $e_2^{oo} = 0$.

  - √ The combination $(e_1^{oo} = e_1^{**}, e_2^{oo} = 0)$ is the second-best incentive-contract solution, since at e_1^{**} the marginal disutility of effort equals the marginal utility of the marginal product of effort.

  - √ In that case, in the second-best the principal would push e_1^{oo} above the first-best level.

- The agent <u>does not</u> substitute between the task with easy-to-measure output and the task with hard-to-measure output, but between <u>each task</u> and <u>leisure</u>.

  - The best the principal can do may be to <u>ignore the multitasking feature</u> of the problem and just get the incentives right for the task whose output he <u>can</u> measure.

- A monitoring contract

  - The <u>first-best</u> effort levels <u>can</u> be attained.

  - The monitoring contract might not even be superior to the second-best incentive contract if the monitoring cost $\bar{C}$ were too big.

    - √ But monitoring <u>can</u> induce any level of e_2 the principal desires.

- The base wage may even be <u>negative</u>,

  which can be interpreted

  - √ as a <u>bond</u> for good effort posted by the agent or

  - √ as <u>a fee</u> he pays for the privilege of filling the job and

    possibly earning m_1 or m_2.

- The agent will choose e_1 and e_2 to maximize

  $$\pi_{agent} = \bar{m} + e_1 m_1 + e_2 m_2 - e_1^2 - e_2^2$$

  subject to $e_1 + e_2 \leq 1$.

  - √ The principal finds the agent working on Task *i* with probability e_i.

  - √ $$\partial \pi_{agent} / \partial e_1 = m_1 - 2e_1 = 0$$

    $$\partial \pi_{agent} / \partial e_2 = m_2 - 2e_2 = 0$$

- The principal will pick m_1^{**} and m_2^{**} to induce the agent to choose e_1^{**} and e_2^{**}.

  √ $m_1^{**} = 2e_1^{**}$

  $m_2^{**} = 2e_2^{**}$

- The base wage $\bar{m}$

  √ the binding participation constraint

$$\pi_{agent} = \bar{m} + e_1^{**}m_1^{**} + e_2^{**}m_2^{**} - (e_1^{**})^2 - (e_2^{**})^2$$

$$= \bar{m} + 2w^{**} - w^{**} = 0$$

- √ $\bar{m} = -w^{**}$

- √ If the principal finds the agent shirking when he monitors, he will pay the agent an amount of $-w^{**}$.

- √ In the case where $e_1^{**} + e_2^{**} < 1$, the result is surprising because the principal wants the agent to take some leisure in equilibrium.

- √ In the case where $e_1^{**} + e_2^{**} = 1$, the result is intuitive.

- √ The key is that the base wage is important only for inducing the agent to take the job and has no influence whatsoever on the agent's choice of effort.