9.2 Adverse Selection under Certainty: Lemons I and II

- ♦ The principal contracts to buy from the agent a car whose quality is noncontractible despite the lack of uncertainty.

- ♦ The Basic Lemons Model
    - ○ Players
        - √ a buyer and a seller

- The order of play

0 Nature chooses quality type θ for the seller according to the distribution $F(\theta)$.

The seller knows θ,

but while the buyer knows F, he does not know the θ of the particular seller he faces.

1 The buyer offers a price P.

2 The seller accepts or rejects.

- Payoffs

  - √ If the buyer rejects the offer, both players receive payoffs of zero.

  - √ Otherwise, $\pi_{buyer} = V(\theta) - P$ and $\pi_{seller} = P - U(\theta)$,

    where V and U will be defined later.

- The payoffs of both players are <u>normalized</u> to zero

  if no transaction takes place.

  - √ The payoff functions show <u>changes</u> from that base.

- ○ Competition between buyers

- ○ It will often be convenient to discuss the game as if it had many sellers.

  - √ There is a population of sellers of different types,

    one of whom is drawn by Nature to participate in the game.

- ♦ A theme running through all four Lemons models is that when quality is unknown to the buyer, less trade occurs.

- Lemons I: Identical Tastes, Two Types of Sellers
  - Specific functional forms
    - √ quality type $\theta \in \{2{,}000, 6{,}000\}$
    - √ $F(\theta)$ puts probability 0.5 on the first type, $\theta = 2{,}000$, and probability 0.5 on the second type, $\theta = 6{,}000$.
    - √ A payoff profile of $(0, 0)$ will represent the status quo, in which the buyer has \$50,000 and the seller has the car.
    - √ the players' valuations for a car of quality θ
    - √ $\pi_{buyer} = \theta - P$ and $\pi_{seller} = P - \theta$

- If he could <u>observe</u> quality at the time of his purchase,

  the buyer would be willing to pay $6,000 for a good car

  and $2,000 for a lemon.

- The buyer <u>cannot</u> observe <u>quality</u>.

  - √ Assume that he <u>cannot</u> enforce a contract based on his discovery once the purchase is made.

  - √ The buyer would be willing to pay $4,000,

    a <u>price</u> equal to the <u>average quality</u> of cars offered for sale,

    for a car of <u>unknown</u> quality if <u>all cars</u> were on the market.

- √ The buyer would refuse to pay more than $2,000.

- √ Half of the cars are traded in equilibrium, all of them <u>lemons</u>.

- ○ The outcome that half the cars are held off the market is interesting since half the cars do have genuinely <u>higher</u> quality.

- Lemons II: Identical Tastes, a Continuum of Types of Sellers

  - Specific functional forms

    - The quality types are <u>uniformly</u> distributed between 2,000 and 6,000.

    - $F(\theta) = [(\theta - 2{,}000)/4{,}000]\, I_{[2{,}000,\ 6{,}000]}(\theta) + I_{(6{,}000,\ \infty)}(\theta)$, where $I_Z(\cdot)$ is the <u>indicator</u> function of a set Z

    - the players' <u>valuations</u> for a car of <u>quality</u> θ

    - $\pi_{buyer} = \theta - P$ and $\pi_{seller} = P - \theta$

- The probability density function of the continuous uniform distribution is:

$$f(x) = \begin{cases} 1/(b-a) & \text{for } a \le x \le b \\ 0 & \text{for } x < a \text{ or } x > b. \end{cases}$$

The cumulative distribution function of the uniform distribution is:

$$F(x) = \begin{cases} 0 & \text{for } x < a \\ (x-a)/(b-a) & \text{for } a \le x \le b \\ 1 & \text{for } x > b. \end{cases}$$

The mean of the uniform distribution is:

$$E(X) = (a+b)/2.$$

- The average quality is $\bar{\theta} = 4{,}000.$

  - √ The buyer would be willing to pay $\$4{,}000$ for a car of unknown quality if all cars were on the market.

  - √ the average quality of cars offered for sale

- The unravelling continues until the price reaches its equilibrium level of $\$2{,}000.$

  - √ But at $P = 2{,}000,$ the number of cars on the market is infinitesimal.

  - √ The market is completely collapsed!

- Figure 9.2

  - √ the price P of used cars on the vertical axis

  - √ the average quality $\overline{\theta}$ of cars offered for sale on the horizontal axis

  - √ Each price leads to a different average quality, $\overline{\theta}(P)$, and

    the slope of $\overline{\theta}(P)$ is greater than one:

    $$\overline{\theta}(P) = (2{,}000 + P)/2.$$

  - √ If the price rises, the quality of the marginal car offered for sale

    equals the new price,

    but the average quality of cars offered for sale is much lower.

- √ The buyer would be willing to pay a price equal to the average quality of cars offered for sale:

$$P(\bar{\theta}) = \bar{\theta}.$$

- √ In equilibrium, the average quality must equal the price, and the quality of the marginal car offered for sale must equal the price.

- √ the players' valuations for a car of quality θ

- √ At the intersection of the two lines (or curves), these equilibrium conditions are met.

√ The only <u>intersection</u> is the point ($2,000, 2,000).

√ The <u>equilibrium</u> lies on the 45^{o} line through the origin.

♦ There is <u>no efficiency loss</u> in either Lemons I or Lemons II.

○ Since all the players have <u>identical</u> tastes,

it does <u>not</u> matter who ends up owning the cars.

9.3 Heterogeneous Tastes: Lemons III and IV

- Lemons III: Buyers Value Cars More Than Sellers
  - Specific functional forms
    - √ The quality types are <u>uniformly</u> distributed between 2,000 and 6,000.
    - √ $F(\theta) \ = \ [(\theta - 2{,}000)/4{,}000] \, I_{[2{,}000,\ 6{,}000]}(\theta) \ + \ I_{(6{,}000,\ \infty)}(\theta)$,

      where $I_Z(\,\cdot\,)$ is the <u>indicator</u> function of a set Z

- √ Sellers value their cars at exactly their qualities, but buyers have valuations 20 percent greater.

- √ the players' valuations for a car of quality θ

- √ $\pi_{buyer} = 1.2\theta - P$ and $\pi_{seller} = P - \theta$

- √ The buyers outnumber the sellers.

- Figure 9.3

  - √ $\bar{\theta}(P) = (2{,}000 + P)/2$

  - √ The buyer would be willing to pay a price equal to 1.2 times the average quality of cars offered for sale:

    $$P(\bar{\theta}) = 1.2\,\bar{\theta}.$$

  - √ In equilibrium, 1.2 times the average quality must equal the price, and the quality of the marginal car offered for sale must equal the price.

- √ the players' valuations for a car of quality θ

- √ At the intersection of the two lines (or curves), these equilibrium conditions are met.

- √ They intersect only at $(\overline{\theta}, P) = (2{,}500,\ 3{,}000)$.

- Because buyers are willing to pay a premium,

  we only see <u>partial</u> adverse selection.

- The equilibrium is <u>partially</u> pooling.

- In equilibrium, the <u>sellers</u> will capture the gains from trade.

- The outcome is <u>inefficient</u>.

  - √ In a world of perfect information, all the cars would be owned by the "buyers," who value them <u>more</u>.

  - √ Under adverse selection, the <u>buyers</u> only end up owning the low-quality cars.

- Lemons IV: Sellers' Valuations Differ

  - Specific functional forms

    - We model sellers as consumers whose valuations of quality have changed since they bought their cars.

    - the players' valuations for a car of quality θ

    - $\pi_{buyer} = \theta - P$ and $\pi_{seller} = P - (1 + \epsilon)\theta$

    - The random disturbance ϵ can be either positive or negative and has an expected value of zero.

    - The buyers outnumber the sellers.

- Figure 9.4

  - ✓ the <u>average quality</u> of cars offered for sale at price P

    $\overline{\theta}(P) = E(\theta|(1+\epsilon)\theta \leq P)$

  - ✓ If $P \geq 6{,}000$, <u>some</u> car owners would be <u>reluctant</u> to sell, because they received <u>positive</u> disturbances to their valuations.

  - ✓ The <u>average quality</u> of cars on the market is less than 4,000 even at $P = 6{,}000$.

- √ Even if $P = 2{,}000$, <u>some</u> sellers with low-quality cars and <u>negative</u> realizations of the disturbance do sell, so the <u>average quality</u> remains above $2{,}000$.

- √ $P(\bar{\theta}) = \bar{\theta}$

- √ In equilibrium, the <u>average quality</u> must equal the <u>price</u>, and the <u>marginal seller's</u> valuation $(1 + \epsilon)\theta_m$ for his car offered for sale must equal the <u>price</u>.

- √ the players' <u>valuations</u> for a car of <u>quality</u> θ

- √ At the <u>intersection</u> of the two curves, these equilibrium conditions are met.

- ♦ A <u>theme</u> running through all four Lemons models is that when <u>quality</u> is <u>unknown</u> to the buyer, <u>less trade occurs</u>.

- More Sellers Than Buyers
  - Lemons III
    - √ $\pi_{buyer} = 1.2\theta - P$ and $\pi_{seller} = P - \theta$
    - √ The buyers <u>outnumber</u> the sellers.
    - √ A <u>buyer</u> would offer a <u>higher price</u> to purchase a car.
    - √ The <u>sellers</u> earn producer surplus.
    - √ The market clears.

- The sellers <u>outnumber</u> the buyers.

  - √ If there were <u>enough sellers</u> with quality $\theta = 2{,}000$, each buyer would pay $P = \$2{,}000$ for a car worth $2{,}400$ to him, acquiring a surplus of 400.

  - √ If there were <u>fewer sellers</u>, the equilibrium price would be <u>higher</u> and some sellers would receive producer surplus.

- Heterogeneous Buyers: Excess Supply

  - Lemons III

    - $\pi_{buyer} = 1.2\theta - P$ and $\pi_{seller} = P - \theta$
    - The buyers <u>outnumber</u> the sellers.
    - A <u>buyer</u> would offer a <u>higher price</u> to purchase a car.
    - The <u>sellers</u> earn producer surplus.
    - The market clears.

  - If buyers have <u>different valuations</u> for a car of <u>quality</u> θ,

    then the market might <u>not</u> clear.

- Risk Aversion
  - Lemons III
    - √ $\pi_{buyer} = 1.2\theta - P$ and $\pi_{seller} = P - \theta$
    - √ The buyers <u>outnumber</u> the sellers.
    - √ A <u>buyer</u> would offer a <u>higher price</u> to purchase a car.
    - √ The <u>sellers</u> earn producer surplus.
    - √ The market clears.

- The buyers and sellers are both <u>risk-averse</u>.

  - √ The <u>seller</u> runs <u>no risk</u>.

  - √ The <u>buyer</u> does bear <u>risk</u>,

    because he buys a car of <u>uncertain quality</u>.

  - √ The utility increased from adding 500 quality units would be

    <u>less</u> than the utility decreased from subtracting 500.

  - √ The <u>equilibrium</u> has a <u>lower</u> price and a <u>lower</u> average quality.