9.4 Adverse Selection under Uncertainty: Insurance Game III

- A firm's customers are "adversely selected" to be accident-prone.

- Insurance Game III
  - Players
    - Smith and two insurance companies

- The order of play

0 Nature chooses Smith to be either *Safe*, with probability 0.6, or *Unsafe*, with probability 0.4.
Smith knows his type, but the insurance companies do not.

1 Each insurance company offers its own contract (x, y) under which Smith pays premium x unconditionally and receives compensation y if there is a theft.

2 Smith picks a contract.

3 Nature chooses whether there is a theft, using probability 0.5 if Smith is *Safe* and 0.75 if he is *Unsafe*.

- Payoffs

  - ✓ Smith's payoff depends on his type and the contract (x, y) that he accepts.

    Assume that $U' > 0$ and $U'' < 0$.

$$\pi_{Smith}\,(Safe) = 0.5\,U(12 - x) + 0.5\,U(0 + y - x)$$

$$\pi_{Smith}\,(Unsafe) = 0.25\,U(12 - x) + 0.75\,U(0 + y - x)$$

√ The companies' payoffs depend on what types of customers accept their contracts.

Company payoff	*Types of customers*
0	No customers
$0.5\,x + 0.5\,(x - y)$	Just *Safe*
$0.25\,x + 0.75\,(x - y)$	Just *Unsafe*
$0.6\,[0.5\,x + 0.5\,(x - y)] + 0.4\,[0.25\,x + 0.75\,(x - y)]$	*Unsafe* and *Safe*

- Figure 9.5

  - The insurance company is risk-neutral,

    so its indifference curve is a straight line with negative slope.

  - Smith's indifference curves

    - √ the slope of an indifference curve

$$p_1 u(x_1) + p_2 u(x_2) = k$$

$$\text{Slope} = dx_2/dx_1 = -p_1 u'(x_1)/p_2 u'(x_2) < 0$$

$$d(\text{Slope})/dx_1 \equiv d^2 x_2/dx_1^2 = -p_1 u''(x_1)/p_2 u'(x_2)$$
$$+ [p_1 u'(x_1) u''(x_2)/p_2 (u'(x_2))^2]\,(dx_2/dx_1) > 0$$

- √ Smith is <u>risk-averse</u>, so his indifference curves are <u>convex</u>.

- √ At any point, the <u>slope</u> of the solid (*Safe*) indifference curve is <u>steeper</u> than that of the dashed (*Unsafe*) indifference curve.

○ No <u>pooling</u> equilibrium exists.

- √ Since the <u>slopes</u> of the dashed and solid indifference curves <u>differ</u>, we can <u>insert</u> another contract, C_2, between them and just barely to the right of ωF.

- √ The <u>attraction</u> of the *Safe* customers away from pooling is referred to as <u>cream skimming</u>, although profits are still zero when there is <u>competition</u> for the cream.

- Figure 9.6

  - Consider whether a separating equilibrium exists.

  - To avoid attracting *Unsafe*s,

    the *Safe* contract must be below the *Unsafe* indifference curve.

  - Contract C_5 is the fullest insurance the *Safe*s can get

    without attracting *Unsafe*s.

    - √ It satisfies the self-selection and competition constraints.

- Figure 9.7

    - Contract C_5, however, might not be an equilibrium either.

    - If one firm offered C_6,

        it would attract both types, *Unsafe* and *Safe*, away from C_3 and C_5,

        because it is to the right of the indifference curves passing through

        those points.

    - Would C_6 be profitable?

    - No equilibrium whatsoever exists.

9.5 Market Microstructure

- This is adverse selection,

  because the informed trader has better information

  on the value of the stock, and

  no uninformed trader wants to trade with an informed trader.

  - The informed trader is a "bad type" from the point of view of

    the other side of the market.

- An <u>institution</u> that many markets have developed is

    the "<u>marketmaker</u>" or "<u>specialist</u>,"

    a trader in a particular stock who is <u>always</u> willing to buy or sell

    to keep the market <u>going</u>.

- This just transfers the <u>adverse selection</u> problem to the marketmaker,

    who <u>always</u> loses

    when he trades with someone who is <u>informed</u>.

- The two models

  - In the Bagehot model,

    there may or may not be one or more informed traders,

    but the informed traders as a group have a trade of fixed size

    if they are present.

  - The marketmaker must decide how big a bid-ask spread to charge.

- In the Kyle model,

  there is one informed trader, who decides how much to trade.

- On observing the imbalance of orders,

  the marketmaker decides what price to offer.

- The Kyle model focuses on the decision of the informed trader,

  not the marketmaker.

♦ The Bagehot Model

○ Players

√ the informed trader and two competing marketmakers

○ The order of play

0 Nature chooses the asset value v

to be either $p_0 - \delta$ or $p_0 + \delta$ with equal probability.

The marketmakers never observe the asset value,

nor do they observe whether anyone else observes it,

but the "informed" trader observes v with probability θ.

1 The marketmakers choose their spreads s, offering prices $p_{bid} = p_0 - s/2$ at which they will buy the security and $p_{ask} = p_0 + s/2$ for which they will sell it.

2 The informed trader decides whether to buy one unit, sell one unit, or do nothing.

3 Noise traders buy n units and sell n units.

- Payoffs

  - Everyone is risk-neutral.

  - The informed trader's payoff is

    $(v - p_{ask})$ if he buys,

    $(p_{bid} - v)$ if he sells, and zero if he does nothing.

  - The marketmaker who offers the highest p_{bid} trades with all the customers who wish to sell.

  - The marketmaker who offers the lowest p_{ask} trades with all the customers who wish to buy.

- √ If the marketmakers set equal prices, they split the market evenly.

- √ A marketmaker who sells x units gets a payoff of $x(p_{ask} - v)$, and a marketmaker who buys x units gets a payoff of $x(v - p_{bid})$.

- Optimal strategies

  - Competition between the marketmakers will make their prices identical and their profits zero.

  - The informed trader should buy if $v > p_{ask}$ and sell if $v < p_{bid}$.

    - √ He has no incentive to trade if $v \in [p_{bid},\ p_{ask}]$.

  - A marketmaker's total expected profit from sales at the ask price of $(p_0 + s/2)$

    - √ The noise traders always buy n units.

- √ The informed trader will buy <u>nothing</u>

  if the true value of the stock is $(p_0 - \delta)$.

- √ The informed trader will <u>buy</u> one unit

  if the true value of the stock is $(p_0 + \delta)$.

- √ The expected value of the stock is p_0.

- √ The informed trader <u>observes</u> the true value with probability θ.

- √ A marketmaker's <u>expected profit</u> is

$$0.5\, n\, [(p_0 + s/2) - (p_0 - \delta)]$$

$$+\ 0.5\,(n + \theta)\,[(p_0 + s/2) - (p_0 + \delta)],$$

where $\delta > s/2$.

- √ If $s > 0$,

the marketmakers will <u>make money</u> dealing with the noise traders

but <u>lose money</u> with the informed trader, if he is present.

- A marketmaker's total expected profit from sales

  at the ask price of $(p_0 + s/2)$

  must be zero.

  - √ $s^* = 2\delta\theta/(2n + \theta)$

- A marketmaker's total expected profit from purchases

  at the bid price of $(p_0 - s/2)$

  must be zero.

  - √ $s^* = 2\delta\theta/(2n + \theta)$

- Implications of s^*

  - √ The spread s^* is positive,

    so that the bid price and the ask price are different.

  - √ $\partial s^* / \partial \delta > 0$ because divergent true values increase losses

    from trading with the informed trader.

  - √ $\partial s^* / \partial n < 0$ because when there are more noise traders,

    the profits from trading with them are greater.

  - √ $\partial s^* / \partial \theta > 0$

- ♦ The Kyle Model

  - ○ Players

    - √ the informed trader and <u>two</u> competing marketmakers

  - ○ The order of play

    0 <u>Nature</u> chooses the <u>asset value</u> v from a <u>normal</u> distribution with mean p_0 and variance σ_v^2, <u>observed</u> by the informed trader but <u>not</u> by the marketmakers.

1 The informed trader offers a trade of size $x(v)$,

which is a purchase if positive and a sale if negative,

unobserved by the marketmaker.

2 Nature chooses a trade of size u by noise traders,

unobserved by the marketmaker,

where u is distributed normally with mean zero and variance σ_u^2.

3 The marketmakers observe the total market trade offer

$y = x + u$, and choose prices $p(y)$.

4 Trades are executed.

If y is positive (the market wants to purchase, in net), whichever marketmaker offers the lowest price executes the trades.

If y is negative (the market wants to sell, in net), whichever marketmaker offers the highest price executes the trades.

The value v is then revealed to everyone.

- Payoffs

  - √ All players are risk-neutral.

  - √ The informed trader's payoff is $(v - p)x$.

  - √ The marketmaker's payoff is zero if he does not trade and $(p - v)y$ if he does.

- An equilibrium for this game is the strategy profile

$$x(v) = (v - p_0)\,(\sigma_u/\sigma_v)$$

and

$$p(y) = p_0 + (\sigma_v/2\sigma_u)y.$$

  - If σ_v^2/σ_u^2 is large,

    then the asset value fluctuates more than the amount of

    noise trading, and

    it is difficult for the informed trader to conceal his trades

    under the noise.

- √ The informed trader will trade <u>less</u>.

- √ A given amount of trading will cause a <u>greater</u> response from the marketmaker.

- √ A trade of given size will have a <u>greater</u> impact on the price.

○ A unique <u>linear</u> equilibrium (but not a unique equilibrium)

- The Bagehot model is perhaps a better explanation of why marketmakers might charge a bid-ask spread even under competitive conditions and with zero transactions costs.

  - Its assumption is that the marketmaker cannot change the price depending on volume, but must instead offer a price, and then accept whatever order comes along.

9.6 A Variety of Applications

- Price Dispersion
- Health Insurance
- Henry Ford's Five-Dollar Day
- Bank Loans
- Solutions to Adverse Selection

9.7 Adverse Selection and Moral Hazard Combined: Production Game VII

♦ Production Game VII: Adverse Selection and Moral Hazard

- Players
  - √ the principal and the agent
- The order of play

0 Nature chooses the state of the world s, observed by the agent but not by the principal, according to distribution $F(s)$, where the state s is Good with probability 0.5 and Bad with probability 0.5.

1 The principal offers the agent a wage contract $w(q)$.

2 The agent accepts or rejects the contract.

3 If the agent accepts, he chooses effort level e.

4 Output is $q = q(e, s)$, where $q(e, good) = 3e$ and $q(e, bad) = e$.

- Payoffs
  - √ If the agent rejects the contract,

    then $\pi_{agent} = \bar{U} = 0$ and $\pi_{principal} = 0$.

  - √ Otherwise, $\pi_{agent} = U(e, w, s) = w - e^2$ and

    $\pi_{principal} = V(q - w) = q - w$.

- Thus, there is no uncertainty,

  both principal and agent are risk-neutral in money, and

  effort is increasingly costly.

- The principal cannot observe effort, but can observe output.

- The first-best effort depends on the state of the world.

  - The principal can observe the state of the world and

    the agent's effort level.

  - In the good state, the social surplus maximization problem is

    $$\underset{e_g}{Maximize} \quad 3e_g - e_g^2.$$

    - √ the optimal effort $e_g^* = 1.5$

    - √ $q_g^* = 4.5$

- In the bad state, the social surplus maximization problem is

$$\underset{e_b}{Maximize} \quad e_b - e_b^2.$$

  - √ the optimal effort $e_b^* = 0.5$
  - √ $q_b^* = 0.5$

- The problem is that the principal does not know what level of effort and output are appropriate.

  - The principal does not want to require high output in both states,

    because if he does,

    he will have to pay too high a salary to the agent

    to compensate for the difficulty of attaining that output

    in the bad state.

- To design the <u>second-best</u> contract,

  he must solve the following problem:

$$\underset{q_g,\, q_b,\, w_g,\, w_b}{\text{Maximize}} \quad [0.5(q_g - w_g) + 0.5(q_b - w_b)]$$

such that

√ the agent has a choice between two <u>forcing contracts</u>,

(q_g, w_g) and (q_b, w_b),

and

√ the contracts must induce <u>participation</u> and <u>self selection</u>.

- The <u>self-selection</u> constraints

  - √ in the <u>good</u> state

$$\pi_{agent}(q_g, w_g \mid good) = w_g - (q_g/3)^2 \tag{9.21}$$

$$\geq w_b - (q_b/3)^2 = \pi_{agent}(q_b, w_b \mid good)$$

  - √ in the <u>bad</u> state

$$\pi_{agent}(q_b, w_b \mid bad) = w_b - q_b^2 \tag{9.22}$$

$$\geq w_g - q_g^2 = \pi_{agent}(q_g, w_g \mid bad)$$

- The <u>participation</u> constraints

  - √ in the <u>good</u> state

$$\pi_{agent}(q_g, w_g \mid good) \; = \; w_g - (q_g/3)^2 \; \geq \; 0 \tag{9.23}$$

  - √ in the <u>bad</u> state

$$\pi_{agent}(q_b, w_b \mid bad) \; = \; w_b - q_b^2 \; \geq \; 0 \tag{9.24}$$

- The bad state's participation constraint (9.24) will be binding, since in the bad state the agent will not be tempted by the good-state contract's higher output and wage.

  - √ Let constraint (9.22) not be binding.

  - √ $w_b = q_b^2$

- The good state's participation constraint (9.23) will not be binding.
  - √ Otherwise, constraint (9.24) is not satisfied due to constraint (9.21).
  - √ If constraint (9.24) is satisfied, then $w_g - (q_g/3)^2 > 0$ due to constraint (9.21).
  - √ The principal must leave the agent some surplus to induce him to reveal the good state.
  - √ an informational rent

- The good state's <u>self-selection</u> constraint (9.21) will be <u>binding</u>.

  - √ In the <u>good</u> state, let the agent be <u>tempted</u> to take the <u>easier</u> contract appropriate for the bad state.

  - √ $w_g - (q_g/3)^2 = w_b - (q_b/3)^2$

    $w_g = (q_g/3)^2 + q_b^2 - (q_b/3)^2$

- The bad state's <u>self-selection</u> constraint (9.22) will <u>not</u> be binding.

  - √ Let the agent <u>not</u> be tempted to produce a large amount for a large wage.

  - √ $w_b - q_b^2 > w_g - q_g^2$

  - √ Solve the <u>relaxed problem</u> without this constraint, and then check that this <u>constraint</u> is indeed satisfied.

- The <u>second-best</u> contract

  - The principal's maximization problem rewritten

    $$\underset{q_g,\, q_b}{\text{Maximize}} \quad [0.5\{q_g - (q_g/3)^2 - q_b^2 + (q_b/3)^2\} + 0.5(q_b - q_b^2)]$$

    - ✓ Eliminate w_b and w_g from the maximand using the two binding constraints, and perform the unconstrained maximization.

  - $q_g^{**} = 4.5 \qquad q_b^{**} \approx 0.26$

    $w_g^{**} \approx 2.32 \qquad w_b^{**} \approx 0.07$

- The bad state's <u>self-selection</u> constraint (9.22) is <u>satisfied</u>.

  √ $w_b^{**} - (q_b^{**})^2 > w_g^{**} - (q_g^{**})^2$

- In the second-best world of information asymmetry,

  the effort in the <u>good</u> state remains at the <u>first-best</u> effort,

  but the <u>second-best</u> effort in the <u>bad</u> state is <u>lower</u> than

  the <u>first-best</u> effort.

  √ <u>Bad-state</u> output and compensation must be <u>suppressed</u>.

  √ <u>Good-state</u> output should be left at the <u>first-best</u> level,

  since the agent will <u>not</u> be tempted by that contract in the bad state.

- In the good state, the agent earns an informational rent.

  - √ This is because the good-state agent could always earn

    a positive payoff

    by pretending the state was bad and taking that contract,

    so any contract that separates out the good-state agent

    (while leaving some contract acceptable to the bad-state agent)

    must also have a positive payoff.

- Such adverse selection problems can be easily solved step-by-step as follows.

  - √ Bolton and Dewatripont (2005)

- ○ Step 1

  Apply the revelation principle.

  - √ Without loss of generality, we can restrict each schedule $T(q)$ to the pair of optimal choices made by the two types of buyers $\{[T(q_L), q_L]$ and $[T(q_H), q_H]\}$.

  - √ This restriction also simplifies greatly the incentive constraints.

- Step 2

  Observe that the participation constraint of the "high" type will not bind at the optimum.

- Step 3

  Solve the relaxed problem without the incentive constraint that is satisfied at the first-best optimum.

- Step 4

  Observe that the two remaining constraints of the relaxed problem will bind at the optimum.

- <u>Step 5</u>

Eliminate T_L and T_H from the maximand

using the two <u>binding</u> constraints,

perform the unconstrained optimization,

and then check that (ICL) is indeed <u>satisfied</u>.

√ This <u>interior</u> solution implies $q_L^* < q_H^*$.

√ One can then immediately verify that the <u>omitted</u> constraints

are <u>satisfied</u> at the optimum $(q_i^*, T_i^*,\ i = L, H)$

given that (ICH) <u>binds</u>.