Chapter 10 Mechanism Design and Postcontractual Hidden Knowledge

10.1 Mechanisms, Unravelling, Cross Checking, and the Revelation Principle

- A <u>mechanism</u> is a <u>set of rules</u> that one player constructs and another freely accepts in order to convey <u>information</u> from the second player to the first.

  - The mechanism contains an <u>information report</u> by the second player and a <u>mapping</u> from each possible report to some <u>action</u> by the first.

- Adverse selection models can be viewed as problems of mechanism design.
  - The contract offers are a mechanism for getting the agents to truthfully report their types.
- Mechanism design goes beyond simple adverse selection.
  - It can be useful even when players begin a game with symmetric information or when both players have hidden information that they would like to exchange.

Postcontractual Hidden Knowledge

- Moral hazard games

  - √ complete information

- Moral hazard with hidden knowledge
  (also called postcontractual adverse selection)

  - √ symmetric information at the time of contracting

  - √ asymmetric information after a contract is signed

  - √ The principal's concern is to give agents incentives to disclose their types later.

- √ The <u>participation</u> constraint is based on the agent's <u>expected</u> payoffs across the different <u>types</u> of agent he might become.

- √ There is <u>just one</u> participation constraint even if there are <u>eventually</u> *n* possible types of agents in the model, rather than the *n* participation constraints that would be required in a <u>standard</u> adverse selection model.

- What makes postcontractual hidden knowledge an <u>ideal setting</u> for the paradigm of <u>mechanism design</u> is that the problem is to set up a contract that

  - √ induces the agent to make a <u>truthful report</u> to the principal, and
  - √ is <u>acceptable</u> to both the principal and the agent.

- ♦ Production Game VIII: Mechanism Design

  - ○ Players

    - √ the principal and the agent

  - ○ The order of play

    1. The principal offers the agent a wage contract of the form $w(q, m)$, where q is output and m is a message to be sent by the agent.

    2. The agent accepts or rejects the principal's contract.

3 Nature chooses the state of the world s, according to probability distribution $F(s)$, where the state s is *good* with probability 0.5 and *bad* with probability 0.5.

The agent observes s, but the principal does not.

4 If the agent accepted, he exerts effort e unobserved by the principal, and sends message $m \in \{good, bad\}$ to him.

5 Output is $q(e, s)$, where $q(e, good) = 3e$ and $q(e, bad) = e$, and the wage is paid.

- Payoffs

  - √ If the agent rejects the contract,

    then $\pi_{agent} = \bar{U} = 0$ and $\pi_{principal} = 0$.

  - √ If the agent accepts the contract,

    then $\pi_{agent} = U(e, w, s) = w - e^2$ and

    $\pi_{principal} = V(q - w) = q - w$.

- The agent does <u>not</u> know his type at the point in time at which he must accept or reject the contract.

- The message m is <u>cheap talk</u> – it does not affect payoffs directly and there is no penalty for lying.

- The principal <u>cannot</u> observe effort, but can observe <u>output</u>.

- The principal implements a mechanism to extract the agent's information.

  - In noncooperative games,

    we ordinarily assume that agents have no moral sense.

  - Since the agent's words are worthless, the principal must try to design

    a contract that either provides incentive for truth telling or

    takes lying into account.

- The first-best effort depends on the state of the world.

  - The principal can observe the state of the world and

    the agent's effort level.

  - In the good state, the social surplus maximization problem is

    $$\underset{e_g}{\mathit{Maximize}} \quad 3e_g - e_g^2.$$

    - the optimal effort $e_g^* = 1.5$

    - $q_g^* = 4.5$

- In the bad state, the social surplus maximization problem is

$$\underset{e_b}{Maximize} \quad e_b - e_b^2.$$

  - √ the optimal effort $e_b^* = 0.5$

  - √ $q_b^* = 0.5$

- The optimal contract

  - The optimal contract must satisfy <u>just one</u> participation constraint, with the <u>two</u> incentive compatibility constraints.

  - The principal must solve the problem:

$$\underset{q_g,\, q_b,\, w_g,\, w_b}{\mathit{Maximize}} \quad [0.5\,(q_g - w_g) + 0.5\,(q_b - w_b)] \qquad (10.1)$$

such that

√ the agent is paid under a <u>forcing contract</u>, (q_g, w_g),

if he reports $m = good$, and

under a <u>forcing contract</u>, (q_b, w_b), if he reports $m = bad$,

√ producing a <u>wrong</u> output for a given contract results in

<u>boiling in oil</u>,

and

√ the contracts must induce <u>participation</u> and <u>self selection</u>.

- The self-selection constraints

  - in the good state

$$\pi_{agent}(q_g, w_g \mid good) = w_g - (q_g/3)^2 \tag{10.2}$$

$$\geq w_b - (q_b/3)^2 = \pi_{agent}(q_b, w_b \mid good)$$

  - in the bad state

$$\pi_{agent}(q_b, w_b \mid bad) = w_b - q_b^2 \tag{10.3}$$

$$\geq w_g - q_g^2 = \pi_{agent}(q_g, w_g \mid bad)$$

- The single participation constraint

  - √ At the time of contracting,

    the agent does not know what the state will be.

  - √ $$0.5\,\pi_{agent}(q_g, w_g \mid good) \;+\; 0.5\,\pi_{agent}(q_b, w_b \mid bad) \tag{10.4}$$

    $$=\; 0.5\,\{w_g - (q_g/3)^2\} \;+\; 0.5\,(w_b - q_b^2) \;\geq\; 0.$$

- The single <u>participation</u> constraint (10.4) is <u>binding</u>.

  - √ The principal wants to pay the agent as little as possible.

  - √ $0.5\,\{w_g - (q_g/3)^2\} + 0.5\,(w_b - q_b^2) = 0$

- The good state's <u>self-selection</u> constraint (10.2) will be <u>binding</u>.

  - ✓ In the good state, the agent will be <u>tempted</u>

    to take the <u>easier contract</u> appropriate for the bad state,

    and so the principal has to <u>increase</u> the agent's payoff from

    the good-state contract

    to yield him at least <u>as much as</u> in the bad state.

  - ✓ $w_g - (q_g/3)^2 = w_b - (q_b/3)^2$

- From the two <u>binding</u> constraints, we obtain the following expressions

  for w_b and w_g.

  - ✓ $w_b = (5/9)\, q_b^2$
  - ✓ $w_g = (1/9)\, q_g^2 + (4/9)\, q_b^2$

- The bad state's <u>self-selection</u> constraint (10.3) will <u>not</u> be binding.

  - ✓ Let the agent <u>not</u> be tempted to produce a large amount

    for a large wage.
  - ✓ $w_b - q_b^2 > w_g - q_g^2$
  - ✓ Solve the <u>relaxed problem</u> without this constraint,

    and then <u>check</u> that this constraint is indeed <u>satisfied</u>.

- The second-best contract

  - The principal's maximization problem (10.1) rewritten

$$\underset{q_g,\, q_b}{\textit{Maximize}} \quad [0.5\,\{q_g - (1/9)q_g^2 - (4/9)q_b^2\} + 0.5\,\{q_b - (5/9)q_b^2\}]$$

    - √ Eliminate w_b and w_g from the maximand

      using the two binding constraints, and

      perform the unconstrained maximization.

  - $q_g^{**} = 4.5 \qquad q_b^{**} = 0.5$

    $w_g^{**} \approx 2.36 \qquad w_b^{**} \approx 0.14$

- The bad state's <u>self-selection</u> constraint (10.3) is <u>satisfied</u>.
  - √ $w_b^{**} - (q_b^{**})^2 > w_g^{**} - (q_g^{**})^2$
- Note that, if the <u>realization</u> of the state of the world is the bad state, then the agent's payoff is <u>negative</u>.
  - √ Does a breach of the contract or renegotiation occur?
- In both states, effort is at the <u>first-best</u> level.
- The agent does <u>not</u> earn informational rents.
  - √ At the time of contracting, he has <u>no</u> private information.

- The principal in Production Game VIII is <u>less</u> constrained, compared to Production Game VII,

  and thus able to come <u>closer</u> to the first-best when the state is <u>bad</u>, and <u>reduce</u> the rents to the agent.

Observable but Nonverifiable Information and the Maskin Matching Scheme

- Three players involved in the contracting situation
  - √ the principal who offers the contract
  - √ the agent who accepts it
  - √ the court that enforces it

- We say that the variable s is nonverifiable
  if contracts based on it cannot be enforced.

- What if the state is <u>observable</u> by both the principal and the agent, but is <u>not</u> public information?

  - √ nonverifiable

  - √ Mutual observability can help.

  - √ Maskin (1977) suggests <u>cross checking</u>.

- Cross checking for Production Game VIII

1 Principal and agent simultaneously send messages m_p and m_a to the court saying whether the state is good or bad.

If $m_p \neq m_a$,

then no contract is chosen and both players earn zero payoffs.

If $m_p = m_a$, the court enforces part 2 of the scheme.

2 The agent is paid the wage $(w \mid q)$ with either the good-state forcing contract $(2.25 \mid 4.5)$ or the bad-state forcing contract $(0.25 \mid 0.5)$, depending on his report m_a,

or is boiled in oil if the output is inappropriate to his report.

- √ There exists an <u>equilibrium</u> in which both players are willing to send <u>truthful messages</u>,

  because a deviation would result in zero payoffs.

- √ The agent earns a payoff of <u>zero</u>,

  because the principal has all of the bargaining power.

- √ The principal's payoff is <u>positive</u>,

  and efforts are at the <u>first-best</u> level.

- Usually this kind of scheme has <u>multiple</u> equilibria.

  - √ <u>perverse</u> ones in which both players send <u>false messages</u> which match and <u>inefficient</u> actions result

- A bigger <u>problem</u> than the multiplicity of equilibria is <u>renegotiation</u> due to players' <u>inability</u> to commit to the mechanism.

Unravelling: Information Disclosure When Lying Is Prohibited

- Another special case in which hidden information can be forced
  into the open
  when the agent is prohibited from lying and
  only has a choice between telling the thruth or remaining silent

- Production Game VIII
  - √ $m = bad$ in the bad state
  - √ If $m = silent$, the principal knows the state must be good.
  - √ The option to remain silent is worthless to the agent.

- $s \sim U[0, 10]$

    - √ The agent's payoff is <u>increasing</u> in the principal's estimate of s.

    - √ The agent <u>cannot</u> lie but he <u>can</u> conceal information.

    - √ The principal would continue this process of <u>logical unravelling</u> to conclude that $s = 2$.

    - √ The principal would make the <u>same</u> deduction from $m \geq 2$ as from $m = 2$.

- The unique equilibrium must be <u>fully separating</u>.

  - √ Somebody would deviate from any partially pooling equilibrium.

- Perfect unravelling is <u>paradoxical</u>.

The Revelation Principle

- ○ A principle can design and offer a contract that induces his agent to lie in equilibrium.
  - √ He can take lying into account.
  - √ This complicates the analysis.
- ○ The revelation principle helps us simplify contract design.

- The revelation principle

  - For every contract $w(q, m)$ that leads to lying (i.e., to $m \neq s$),

    there is a contract $w^*(q, m)$ with the same payoff for every s

    but no incentive for the agent to lie.

- There are two levels of simplification in mechanism design.

  - If there are n possible types of agent,

    we can restrict the agent's message to take only n values.

  - We can require the mechanism to be constructed

    to elicit truthful messages from the agent.

- Direct and indirect mechanisms

  - √ If a mechanism restricts the agent's <u>messages</u> to the set of <u>types</u>, it is called a <u>direct</u> mechanism.

  - √ If a mechanism allows <u>more</u> possible <u>messages</u> than <u>types</u>, it is called a <u>indirect</u> mechanism.

- We can add a <u>third constraint</u> to the incentive compatibility and participation constraints to help calculate the equilibrium.

  - √ <u>truth-telling</u>

    The equilibrium contract makes the agent willing to choose $m = s$.

- The revelation principle depends heavily on the following assumption.

  - √ The principal <u>cannot</u> breach his contract.

- Throughout this chapter, we will be assuming

  that the principal can <u>commit</u> to his mechanism.

  - √ He can <u>commit</u> to not using all the information he receives

    from the agent.

The Sender-Receiver Game of Crawford and Sobel:

Coarse Information Transmission

- Even if the informed and uninformed players have different incentives,

  can lie, and can't commit to a mechanism,

  if their incentives are close enough,

  truthful (if imperfect) messages can be sent in equilibrium.

♦ The Crawford-Sobel Sender-Receiver Game

○ Players

√ the sender (the informed player)

√ the receiver (the uninformed player)

○ The order of play

0 Nature chooses the sender's <u>type</u> to be $t \sim U[0, 10]$.

1 The sender chooses <u>message</u> $m \in [0, 10]$.

2 The receiver chooses <u>action</u> $a \in [0, 10]$.

- Payoffs

  - √ The payoffs are quadratic loss functions in which each player has an ideal point and wants a to be close to that ideal point.

  - √ $\pi_{sender} = \alpha - \{a - (t + 1)\}^2$

  - √ $\pi_{receiver} = \alpha - (a - t)^2$

- Equilibria

  - There is no fully separating equilibrium

    in which each type of sender reports a different message.

    - √ Perfect truthtelling cannot happen in equilibrium.

- Pooling Equilibrium 1

  - √ Sender:

    Send $m = 10$ regardless of t.

  - √ Receiver:

    Choose $a = 5$ regardless of m.

  - √ Out-of-equilibrium belief:

    If the sender sends $m < 10$,

    the receiver uses <u>passive conjectures</u> and

    still believes that $t \sim U[0, 10]$.

- Pooling Equilibrium 2

  - Sender:

    Send m using a <u>mixed-strategy</u> distribution <u>independent</u> of t

    that has the support $[0, 10]$ with positive density everywhere.

  - Receiver:

    Choose $a = 5$ regardless of m.

  - Out-of-equilibrium belief:

    Unnecessary,

    since any message might be observed in equilibrium.

- In each of these two equilibria,

  the sender's message conveys <u>no</u> information, and

  is <u>ignored</u> by the receiver.

- Averaging over all possible *t*,

  both their payoffs are <u>lower</u> than if the sender could commit to

  <u>truthtelling</u>.

- Partial Pooling Equilibrium 3

  - √ Sender:

    Send $m = 0$ if $t \in [0, 3]$ or

    $m = 10$ if $t \in [3, 10]$.

  - √ Receiver:

    Choose $a = 1.5$ if $m < 3$ and $a = 6.5$ if $m \geq 3$.

  - √ Out-of-equilibrium belief:

    If m is something other than 0 or 10,

    then $t \sim U[0, 3]$ if $m \in [0, 3)$ and

    $t \sim U[3, 10]$ if $m \in [3, 10]$.

- In the Sender-Receiver Game,

  the receiver <u>cannot</u> commit to the way he reacts to the message,

  so this is <u>not</u> a mechanism design problem.

  - Instead, this is a <u>cheap-talk game</u>,

    so called because of these absences:

    - √ m does <u>not</u> affect the payoff directly,

    - √ the players <u>cannot</u> commit to future actions, and

    - √ <u>lying</u> brings <u>no</u> direct penalty.

- The sender and the receiver's <u>interests</u> are similar but <u>not</u> identical, and they could both <u>benefit</u> from some <u>transfer</u> of information.

    - If <u>expectations</u> are appropriate, they do so, in the partially pooling equilibrium.

    - If they do <u>not</u> expect the cheap talk to be informative, however, it will not be, and <u>coordination</u> will fail.

10.2 Myerson Mechanism Design

- Depending on <u>who</u> offers the contract and <u>when</u> it is offered, various games result.

- We will look at one in which the <u>seller</u> makes the offer, and does so <u>before</u> he knows whether his quality is high or low.

♦ The Myerson Trading Game: Postcontractual Hidden Knowledge

- Players
  - √ a buyer and a seller

- The order of play

1 The seller offers the buyer a contract $\{q_h, p_h, q_l, p_l\}$ under which the seller will declare his quality m to be high or low, and the buyer will then buy q_l or q_h units of the 100 the seller has available, at price p_l or p_h.

The contract is $\{q(m)p(m), q(m)\}$.

Zero is paid if the wrong output is delivered.

2 The buyer accepts or rejects the contract.

3 Nature chooses whether the type of the seller's good, s, is High quality (probability 0.2) or Low (probability 0.8), unobserved by the buyer.

4 If the contract was accepted by both sides, the seller declares his type to be L or H and sells at the appropriate quantity and price as stated in the contract.

- Payoffs

  - √ If the buyer rejects the contract, $\pi_{buyer} = 0$, $\pi_{seller\ H} = 40 \times 100$, and $\pi_{seller\ L} = 20 \times 100$.

  - √ If the buyer accepts the contract and the seller declares a <u>type</u> that has <u>price</u> p and <u>quantity</u> q, then

$$\pi_{buyer\,|\,L} = (30 - p)\,q,$$

$$\pi_{buyer\,|\,H} = (50 - p)\,q,$$

$$\pi_{seller\ H} = 40\,(100 - q) + pq, \text{ and}$$

$$\pi_{seller\ L} = 20\,(100 - q) + pq.$$

- √ The seller has an <u>opportunity cost</u>

  (a personal value or production cost) of

  40 per high-quality unit and 20 per low-quality unit.

- ○ For <u>efficiency</u>, all of the good should be transferred

  from the seller to the buyer.

- ○ The only way to get the seller to <u>truthfully</u> reveal the quality of

  the good, however, is

  for the buyer to say that if the seller <u>admits</u> the quality is <u>bad</u>,

  he will buy <u>more</u> units than if the seller <u>claims</u> it is <u>good</u>.

- The first-best quantities
  - $q_h^* = 100$ and $q_l^* = 100$

- The optimal contract
  - The seller wants to design a contract subject to two sets of constraints.
  - The participation constraint for the buyer
    - √ $0.8\,\pi_{buyer\,|\,L} + 0.2\,\pi_{buyer\,|\,H} \geq 0$

      $0.8\,(30 - p_l)q_l + 0.2\,(50 - p_h)q_h \geq 0$

- √ This constraint will be binding.

- √ $p_l = 30$ and $p_h = 50$

○ We do not need to write out the seller's participation constraint separately.

- √ the acceptable (if vacuous) null contract

  $\{q_h, p_h, q_l, p_l\} = \{0, 0, 0, 0\}$

- Two <u>incentive compatibility</u> constraints for the <u>seller</u> himself

  √ The <u>seller</u> must design a <u>contract</u> that will induce himself to tell the <u>truth</u> later once he discovers his type.

  √ The seller is trying to sell not just a <u>good</u>, but a <u>contract</u>, and so he must make the contract to be <u>attractive</u> to the buyer.

  √ when he has <u>low</u> quality

$$\pi_{seller\ L}(q_l, p_l) \ \geq \ \pi_{seller\ L}(q_h, p_h)$$

$$20\,(100 - q_l) + 30\, q_l \ \geq \ 20\,(100 - q_h) + 50\, q_h$$

$$q_l \ \geq \ 3q_h \qquad \Rightarrow \qquad q_l \ > \ q_h$$

√ when he has <u>high</u> quality

$$\pi_{seller\ H}(q_h, p_h) \ \geq \ \pi_{seller\ H}(q_l, p_l)$$

$$40\ (100 - q_h) \ + \ 50\ q_h \ \geq \ 40\ (100 - q_l) \ + \ 30\ q_l$$

$$q_h \ \geq \ -\ q_l$$

$\Rightarrow$ satisfied for all possible q_l and q_h

- The seller's maximization problem

  - √ $q_l = 3q_h$ at the optimum

    (from the low-quality incentive compatibility constraint)

  - √ The seller's payoff function

$$\pi_s = 0.8\,\pi_{seller\ L}(q_l, p_l) + 0.2\,\pi_{seller\ H}(q_h, p_h)$$

$$= 0.8\,\{20\,(100 - q_l) + 30q_l\} + 0.2\,\{40\,(100 - q_h) + 50q_h\}$$

√ The seller must solve the problem:

$$\underset{q_l,\, q_h}{\mathit{Maximize}} \quad \{0.8\,(2{,}000 + 10q_l) + 0.2\,(4{,}000 + 10q_h)\}$$

subject to

$$q_l = 3q_h, \quad q_l \leq 100, \quad \text{and} \quad q_h \leq 100.$$

○ $q_h^{**} = 100/3$

$q_l^{**} = 100$

- The <u>equilibrium</u> follows the general <u>pattern</u> for these games.

  - The <u>participation</u> constraint is <u>binding</u> (for the buyer).

  - The <u>incentive compatibility</u> constraint is <u>binding</u>

    for the <u>type</u> with the <u>most temptation</u> to lie,

    and <u>not</u> for the other type.

  - Using the two binding constraints,

    we can solve out for the values of some of the <u>choice variables</u>

    in terms of other choice variables.

  - We can maximize the payoff of the <u>player</u> making the <u>offer</u> (the seller)

    to solve for values of those remaining variables.

- The mechanism will <u>not</u> work

  if further offers <u>can</u> be made <u>after</u> the end of the game.

- The mechanism is <u>not</u> first-best efficient.

- The importance of <u>commitment</u> is a general <u>feature</u> of mechanisms.

- We could have set it up <u>instead</u> as (w, q),

  a total price amount w for the quantity q.

  - That would be <u>more</u> in the style of mechanism design.