10.3 An Example of Postcontractual Hidden Knowledge: The Salesman Game

- If the customer type is a *Pushover*,

  the efficient sales effort is low and sales should be moderate.

- If the customer type is a *Bonanza*,

  the effort and sales should be higher.

♦ The Salesman Game

- Players

  √ a manager and a salesman

- The order of play

1. The manager offers the salesman a contract of the form

   $[w(m),\ q(m)]$,

   where w is the wage, q is sales, and m is a message.

2. The salesman decides whether or not to accept the contract.

3. Nature chooses whether the customer type t is a *Bonanza* or a *Pushover* with probabilities 0.2 and 0.8.

   The salesman observes the type, but the manager does not.

4 If the salesman has accepted the contract,

he chooses his effort e.

His sales level is $q = e$, so his sales perfectly reveal his effort.

5 The salesman's wage is $w(m)$ if he chooses $e = q(m)$

and zero otherwise.

- Payoffs

  - √ The manager is risk-neutral and the salesman is risk-averse.

  - √ If the salesman rejects the contract,

    his payoff is $\bar{U} = 8$ and the manager's is zero.

  - √ If he accepts the contract,

    then $\pi_{manager} = q - w$, and $\pi_{salesman} = U(e, w, t)$,

    where $\partial U/\partial e < 0$, $\partial^2 U/\partial e^2 < 0$, $\partial U/\partial w > 0$,

    and $\partial^2 U/\partial w^2 < 0$.

- The manager can perfectly deduce effort, even out of equilibrium.

♦ The optimal contract

  ○ The manager's indifference curves are straight lines with slope 1.

  ○ The salesman's indifference curves slope upwards, and are convex.

    √ The salesman has two sets of indifference curves,

    solid for *Pushovers* and dashed for *Bonanzas*.

- Figure 10.1

  - √ The optimal truth-telling contract is the pooling contract that pays the intermediate wage of w_3 for the intermediate quantity of q_3, and zero for any other quantity, regardless of the message.

  - √ The pooling contract is a second-best contract, a compromise between the optimum for *Pushovers* and the optimum for *Bonanzas*.

  - √ The contract must satisfy the participation constraint,

    $$0.8\, U(q_3, w_3, Pushover) + 0.2\, U(q_3, w_3, Bonanza) \geq 8.$$

- The <u>nature</u> of the equilibrium depends on the <u>shapes</u> of the indifference curves.

- Figure 10.2

  - √ The equilibrium is <u>separating</u>, not pooling, and there does exist a <u>first-best</u>, <u>fully revealing</u> contract.

  - √ The contract induces the salesman to be <u>truthful</u>, and the <u>incentive compatibility</u> constraints are satisfied.

- The idea is to reward salesmen <u>not</u> just for <u>high</u> effort, but for <u>appropriate</u> effort.

- Another way to look at a <u>separating</u> equilibrium is
  to think of it as a <u>choice</u> of contracts rather than
  as <u>one</u> contract with different <u>wages</u> for different <u>outputs</u>.

  - In this interpretation, the manager offers a <u>menu</u> of contracts and
    the salesman selects <u>one</u> of them
    <u>after</u> learning his <u>type</u>.

- The Salesman Game illustrates a number of ideas.

  - It can have either a pooling or a separating equilibrium.

  - The revelation principle can be applied to avoid having to consider contracts in which the manager must interpret the salesman's lies.

  - It shows how to use diagrams when the algebraic functions are intractable or unspecified.

10.4 The Groves Mechanism

- The principal is an altruistic government that cares directly about the utility of the agents.
  - √ a benevolent government

- The mayor is considering installing a streetlight costing $100.
  - √ He will only install it if he decides that the sum of the residents' valuations for it is greater than or equal to the cost.
  - √ The mayor's problem is to discover their valuations.

♦ The Streetlight Game

○ Players

√ the mayor and <u>five</u> householders

○ The order of play

0 Nature chooses the <u>value</u> v_i

that householder *i* places on having a streetlight installed,

using <u>distribution</u> $f_i(v_i)$.

Only householder *i* <u>observes</u> v_i.

1 The mayor announces a mechanism, M,

which requires a householder who reports m to pay $w(m)$

if the streetlight is installed, and

installs the streetlight

$$\text{if } g(m_1, m_2, m_3, m_4, m_5) \equiv \sum_{j=1}^{5} m_j - 100 \geq 0.$$

2 Householder i reports value m_i simultaneously

with all other householders.

3 If $g(m_1, m_2, m_3, m_4, m_5) \geq 0$,

the streetlight is built and householder i pays $w(m_i)$.

- Payoffs

  - √ The mayor tries to maximize <u>social welfare</u>,

    including the welfare of <u>taxpayers</u> besides the 5 <u>householders</u>.

  - √ His payoff is zero if the streetlight is <u>not</u> built.

  - √ Otherwise, it is

$$\pi_{mayor} = \sum_{j=1}^{5} v_j - 100,$$

subject to the constraint that $\sum_{j=1}^{5} w(m_j) \geq 100$,

so he can raise the taxes to pay for the light.

- √ The payoff of householder i is zero if the streetlight is <u>not</u> built.

- √ Otherwise, it is

$$\pi_i(m_1, m_2, m_3, m_4, m_5) = v_i - w(m_i).$$

- Mechanisms
  - Mechanism M_1
    - √ $\left(w(m_i) = 20,\ Build\ iff\ \sum_{j=1}^{5} m_j \geq 100 \right)$
    - √ Talk is cheap, and

      the dominant strategy would be to overreport or underreport.
    - √ a flawed mechanism

○ Mechanism M_2

√ $$\left(w(m_i) = Max\,\{m_i, 0\},\ Build\ \ iff\ \sum_{j=1}^{5} m_j \geq 100\right)$$

√ If all the householders knew each other's values perfectly, then there would be a continuum of Nash equilibria that attained the efficient result.

√ Each householder would announce up to his valuation if necessary.

- Mechanism M_3

  - √ $$\left(w(m_i) = 100 - \sum_{j \neq i} m_j,\ \mathit{Build\ iff}\ \sum_{j=1}^{5} m_j \geq 100\right)$$

  - √ a Nash equilibrium in which all the players are <u>truthful</u>

  - √ a <u>dominant-strategy</u> mechanism

    - Truthfulness is weakly <u>dominant</u>.
    - The players are strictly better off telling the <u>truth</u> whenever <u>lying</u> would alter the mayor's <u>decision</u>.

  - √ It is <u>not</u> budget-balancing.

  - √ The total tax revenue could easily be <u>negative</u>.

10.5 Price Discrimination

- ○ A problem of <u>mechanism design</u> under adverse selection

♦ Varian's Nonlinear Pricing Game

- ○ Players
  - √ one seller and one buyer
- ○ The order of play

0 Nature assigns the buyer a type, s.

The buyer is "unenthusiastic" with utility function u or "valuing" with utility function v, with equal probability.

The seller does not observe Nature's move, but the buyer does.

1 The seller offers mechanism $\{w_m, q_m\}$ under which the buyer can announce his type as m and buy amount q_m for lump sum w_m.

2 The buyer chooses a message m or rejects the mechanism entirely and does not buy at all.

- Payoffs

  - √ The seller has a <u>zero</u> marginal cost, so his <u>payoff</u> is $w_u + w_v$.

  - √ The buyers' <u>payoffs</u> are $\pi_u = u(q_u) - w_u$ and $\pi_v = v(q_v) - w_v$ if q is positive, and 0 if $q = 0$, with $u', v' > 0$ and $u'', v'' < 0$.

  - √ The marginal willingness to pay is <u>greater</u> for the valuing buyer: for any q,

$$u'(q) < v'(q). \tag{10.27}$$

- Condition (10.27) is an example of the single-crossing property.

  - √ Combined with the assumption that $v(0) = u(0) = 0$, it also implies that $u(q) < v(q)$ for any value of q.

Perfect Price Discrimination

- The game would allow perfect price discrimination

  if the seller did know which buyer had which utility function.

- The seller's maximization problem

  √ $$\underset{q_u,\, q_v,\, w_u,\, w_v}{\text{Maximize}} \quad w_u + w_v$$

  subject to the participation constraints

  - $u(q_u) - w_u \geq 0$
  - $v(q_v) - w_v \geq 0$

- The constraints will be satisfied as equalities.

  - √ $w_u = u(q_u)$
  - √ $w_v = v(q_v)$

- The seller's maximization problem rewritten

  - √ $\underset{q_u,\, q_v}{Maximize} \quad u(q_u) + v(q_v)$

- $u'(q_u^*) = 0 \qquad v'(q_v^*) = 0$

  $w_u^* = u(q_u^*) \qquad w_v^* = v(q_v^*)$

  - √ The entire consumer surpluses are eaten up.

Interbuyer Price Discrimination

- The interbuyer price discrimination problem arises when the seller knows which utility functions Smith and Jones have and can sell to them separately.

- Assume that the seller must charge each buyer a single price per unit and let the buyer choose the quantity.

- The seller's maximization problem

  - √ $$\underset{q_u,\, q_v,\, p_u,\, p_v}{Maximize} \quad p_u q_u + p_v q_v$$

    subject to the <u>participation</u> constraints

    - $u(q_u) - p_u q_u \geq 0$
    - $v(q_v) - p_v q_v \geq 0$

    and the <u>incentive compatibility</u> constraints

    - $q_u = argmax\, [u(q_u) - p_u q_u]$
    - $q_v = argmax\, [v(q_v) - p_v q_v]$

- The buyers' quantity choice problems
  - √ $u'(q_u) - p_u = 0$
  - √ $v'(q_v) - p_v = 0$

- The seller's maximization problem rewritten
  - √ $\underset{q_u,\, q_v}{Maximize} \quad u'(q_u)q_u + v'(q_v)q_v$

    subject to the participation constraints

    - $u(q_u) - u'(q_u)q_u \geq 0$
    - $v(q_v) - v'(q_v)q_v \geq 0$

- ○ The participation constraints will <u>not</u> be binding.
  - √ $u(q_u) - u'(q_u)\, q_u$ is increasing in q_u.
  - √ $v(q_v) - v'(q_v)\, q_v$ is increasing in q_v.

- ○ The first-order conditions
  - √ $u''(q_u)\, q_u \;+\; u'(q_u) \;=\; 0$
  - √ $v''(q_v)\, q_v \;+\; v'(q_v) \;=\; 0$
  - √ two <u>independent</u> problems

- ○ If the <u>cost</u> function were a more general <u>convex</u> function $c(q_u + q_v)$,

  the two first-order conditions would have to be solved <u>together</u>,

  because each condition would depend on both q_u and q_v.

<u>Back to Nonlinear Pricing</u>

- <u>Interquantity</u> price discrimination
  - √ The seller charges different <u>unit prices</u> for different <u>quantities</u>.
- <u>Neither</u> the perfect price discrimination <u>nor</u> the interbuyer problems are <u>mechanism design</u> problems.
  - √ The seller is perfectly <u>informed</u> about the <u>types</u> of the buyers.
- The original game is a problem of <u>mechanism design</u> under adverse selection.
  - √ <u>Separation</u> is the seller's main concern.
  - √ The seller designs incentives to separate the <u>types</u> of the buyers.

- The equilibrium mechanism

  - The seller's maximization problem

    - $$\underset{q_u,\, q_v,\, w_u,\, w_v}{\textit{Maximize}} \quad w_u + w_v$$

      subject to the participation constraints

      - $u(q_u) - w_u \geq 0$
      - $v(q_v) - w_v \geq 0$

      and the self-selection constraints

      - $u(q_u) - w_u \geq u(q_v) - w_v$
      - $v(q_v) - w_v \geq v(q_u) - w_u$

- Not all of these constraints will be binding.

  - √ In a mechanism design problem like this,

    what always happens is that the contracts are designed

    so that one type of agent is pushed down to his reservation utility.

- Suppose that the optimal contract is in fact separating, and

  also that both types accept a contract.

- The unenthusiastic consumer's participation constraint is binding.

  - √ $w_u = u(q_u)$

- The <u>valuing</u> consumer's <u>self-selection</u> constraint is <u>binding</u>.

  √ $w_v = w_u - v(q_u) + v(q_v)$

- The seller's maximization problem <u>reformulated</u>

$$\underset{q_u,\, q_v}{\textit{Maximize}} \quad u(q_u) + u(q_u) - v(q_u) + v(q_v)$$

- The first-order conditions

  √ $u'(q_u) + \{u'(q_u) - v'(q_u)\} = 0$

  √ $v'(q_v) = 0$

- The valuing type buys a quantity such that his last unit's marginal utility exactly equals the marginal cost of production.
  - √ $v'(q_v^{**}) = 0$
  - √ His consumption is at the efficient level.

- The unenthusiastic type buys less than his first-best amount.
  - √ the single-crossing property that $u'(q) < v'(q)$
  - √ $u'(q_u) + \{u'(q_u) - v'(q_u)\} = 0$
  - √ $u'(q_u^{**}) > 0$

- The seller must sell less than first-best optimal

  to the unenthusiastic type

  so as not to make that contract too attractive to the valuing type.

- On the other hand, making the valuing type's contract more valuable

  to him actually helps separation,

  so q_v is chosen to maximize social surplus.

- $q_u^{**} < q_v^{**}$

  √ the single-crossing property that $u'(q) < v'(q)$

  √ $v''(q) < 0$

  √ $u'(q_u^{**}) > 0$ and $v'(q_v^{**}) = 0$

- The equilibrium is <u>separating</u>, not pooling.

- A corner solution

- Despite facing a monopolist,

  the <u>valuing</u> type can end up retaining consumer surplus –

  an <u>informational rent</u>.

  √ a return to his <u>private</u> information about his own type

The Single-Crossing Property

- Condition (10.27) is an example of the single-crossing property, since it implies that the indifference curves of the two agents cross at most one time.

- The valuing buyer has stronger demand than the unenthusiastic buyer.

  √ $u'(q) < v'(q)$ for all q

- Two curves satisfying the single-crossing property

  - √ $u(q) = \sqrt{q}$

  - √ $v(q) = 2\sqrt{q}$

- It is often natural to assume that the single-crossing property holds, and it is a useful sufficient condition for separation to be possible, but it is not a necessary condition.