10.6 Rate-of-Return Regulation and Government Procurement

- The central idea in both government procurement and regulation of natural monopolies is that the government is trying to induce a private firm to efficiently provide a good to the public while covering the cost of production.

- Usually, the firm has better information about costs and demand than the government does.

- The government might <u>auction off</u> the right to provide the service,

  might allow the firm a <u>maximum price</u> (a price cap),

  or might agree to <u>compensate</u> the firm to varying degrees

  for different levels of cost (rate-of-return regulation).

- Procurement I: Full Information

  - Players

    - ✓ the government and the firm

  - The order of play

    0 Nature assigns the firm expensive problems with the project, which add costs of x, with probability θ.

    A firm is thus "normal," with type N and $s = 0$, or "expensive," with type X and $s = x$.

The government and the firm both <u>observe</u> the type.

1 The government offers a <u>contract</u> $\{w(m) = c(m) + p(m),\ c(m)\}$ which pays the firm its <u>observed cost</u> c and a <u>price</u> p if it announces its <u>type</u> to be m and incurs <u>cost</u> $c(m)$, and pays the firm <u>zero</u> otherwise.

2 The firm accepts or rejects the contract.

3 If the firm accepts, it chooses effort level e, <u>unobserved</u> by the government.

4 The firm finishes the missile at a cost of $c = \overline{c} + s - e$,

which is observed by the government,

plus an additional cost of $f(e - \overline{c})$

that the government does not observe.

The government reimburses $c(m)$ and pays $p(m)$.

- Payoffs

  - √ Both firm and government are risk-neutral and both receive payoffs of zero if the firm rejects the contract.

  - √ If the firm accepts, its payoff is

$$\pi_{firm} = p - f(e - \overline{c})$$

where $f(e - \overline{c})$, the cost of effort, is increasing and convex, so $f' > 0$ and $f'' > 0$.

  - √ Assume for technical convenience that f is increasingly convex, so $f''' > 0$.

- √ The government's payoff is

$$\pi_{government} = B - (1+t)\,c - t\,p - f(e - \overline{c})$$

where B is the benefit of the missile and t is the deadweight loss from the taxation needed for government spending.

- ○ If the government cared only about the value of the missile and the cost to taxpayers, its payoff would be

$$\pi_{government} = B - (1+t)\,c - (1+t)\,p.$$

- √ a selfish principal

- The government's payoff is the sum of the welfares of the taxpayers and the firm.

  - √ social welfare

  - √ The fact that taxation has deadweight loss means that the government will want to pay the firm as little as possible.

- Assume that B is large enough that the government definitely wishes to build the missile.

- Cost, not output, is the focus of this model.

- The optimal contract
  - The government can specify a contract conditioned on the type of the firm.
    - √ The government reimburses the firm's costs.
    - √ The government pays p_N to a normal firm with the cost c_N, p_X to an expensive firm with the cost c_X, and $p = 0$ to a firm that does not achieve its appropriate cost level.
    - √ (c_N, p_N) for a normal firm

      (c_X, p_X) for an expensive firm

- The government's <u>maximization problem</u> when the firm is <u>expensive</u>

  - √ $$\underset{c_X,\, p_X}{Maximize} \quad B - (1+t)\, c_X - t\, p_X - f(x - c_X)$$

    subject to the <u>participation</u> constraint

    $$\pi_X(X) = p_X - f(x - c_X) \geq 0$$

    and the <u>incentive compatibility</u> constraint

  - √ In equilibrium, the expensive firm exerts effort $e_X^* = \overline{c} + x - c_X^*$.

  - √ The <u>incentive compatibility</u> constraint is trivial:

    The government can use a <u>forcing contract</u> that pays a firm <u>zero</u>

    if it generates the <u>wrong</u> cost for its type.

- ○ The government's <u>maximization problem</u> when the firm is <u>normal</u>

  - √ $$\underset{c_N,\, p_N}{Maximize} \quad B - (1+t)\, c_N - t\, p_N - f(-c_N)$$

    subject to the <u>participation</u> constraint

    $$\pi_N(N) = p_N - f(-c_N) \geq 0$$

    and the <u>incentive compatibility</u> constraint

  - √ In equilibrium, the normal firm exerts effort $e_N^* = \overline{c} - c_N^*$.

  - √ The government can use a <u>forcing contract</u> that pays a firm <u>zero</u> if it generates the <u>wrong</u> cost for its type.

- To make a firm's payoff zero and

  reduce the deadweight loss from taxation,

  the government will provide prices that do no more than equal

  the firm's disutility of effort.

  - √ $p_X = f(e_X - \overline{c}) = f(x - c_X)$

  - √ $p_N = f(e_N - \overline{c}) = f(- c_N)$

- The government's maximization problem rewritten when the firm is expensive

  - √ $\underset{c_X}{Maximize} \quad B - (1+t)\,c_X - t\,f(x - c_X) - f(x - c_X)$

  - √ the first-order condition

$$f'(x - c_X) = 1$$

- The government's maximization problem rewritten when the firm is normal

  - √ $\underset{c_N}{Maximize} \quad B - (1+t)\,c_N - t\,f(-c_N) - f(-c_N)$

  - √ the first-order condition

$$f'(-c_N) = 1$$

- At the <u>optimal</u> effort level,

  the marginal disutility of effort <u>equals</u> the marginal reduction in cost because of effort.

  √ the <u>first-best</u> efficient effort level e^*

  $$f'(e^* - \overline{c}) = 1$$

  √ <u>Both</u> types exert the <u>same</u> effort level e^*.

- $c_N^* = c_X^* - x$

  - √ $f'(x - c_X^*) = 1$

  - √ $f'(-c_N^*) = 1$

- The <u>cost target</u> assigned to the expensive firm is $c_X^* = \overline{c} + x - e^*$, while that assigned to the normal firm is $c_N^* = \overline{c} - e^*$.

  - √ $c_X^* > c_N^*$

- The two firms exert the <u>same</u> efficient effort level and are paid the <u>same</u> price to compensate for the <u>disutility</u> of effort.

  √ $p_X^* = f(e^* - \overline{c}) = p_N^*$

- B should satisfy

$$B - (1+t)\,c_X^* - t\,f(e^* - \overline{c}) - f(e^* - \overline{c}) \geq 0,$$

which requires

$$B - (1+t)\,(\overline{c} + x - e^*) - (1+t)\,p^* \geq 0.$$

  √ $p^* = p_X^* = p_N^*$

- Summary of results

  - The <u>optimal</u> contracts

    - √ $(c_N^*,\ p_N^*)$ for a normal firm

      $(c_X^*,\ p_X^*)$ for an expensive firm

    - √ forcing contracts

  - The two firms exert the <u>same</u> efficient effort level e^*.

  - <u>Zero</u> profits

    - √ $\pi_X(X^*) = p_X^* - f(x - c_X^*) = 0$

    - √ $\pi_N(N^*) = p_N^* - f(-c_N^*) = 0$

Procurement II: Incomplete Information (Adverse Selection)

- The existence of expensive problems is not observed by the government.

- The cheapest pooling contract

  - If a firm annonces its type to be expensive or normal and

    incurs a cost of $c_P = \overline{c} + x - e^*$,

    then the government pays the firm its observed cost c_P and

    a price of p^*.

  If a firm does not achieve the cost level of $c_P = \overline{c} + x - e^*$,

    then the government pays the firm its observed cost and

    a price of zero.

- It is <u>inefficient</u> because the normal firm can reduce <u>costs</u> to

  $$c_P = \bar{c} + x - e^*$$

  by exerting effort <u>lower</u> than e^*.

- It will turn out that <u>separating</u> contracts will yield <u>higher</u> welfare

  than the pooling contract.

♦ The optimal separating contract

- the contract with the optimal pair of (c_N, p_N) and (c_X, p_X)

○ The government's maximization problem under incomplete information

√ $$\underset{c_N,\, c_X,\, p_N,\, p_X}{\text{Maximize}} \quad \theta\,\{B - (1+t)\,c_X - t\,p_X - f(x - c_X)\}$$

$$+ (1-\theta)\,\{B - (1+t)\,c_N - t\,p_N - f(-c_N)\}$$

√ The <u>participation</u> constraints

- for the expensive firm

$$\pi_X(X) = p_X - f(x - c_X) \geq 0$$

- for the normal firm

$$\pi_N(N) = p_N - f(- c_N) \geq 0$$

√ The <u>incentive compatibility</u> constraints

- for the expensive firm

$$\pi_X(X) = p_X - f(x - c_X) \geq \pi_X(N) = p_N - f(x - c_N)$$

- for the normal firm

$$\pi_N(N) = p_N - f(-c_N) \geq \pi_N(X) = p_X - f(-c_X)$$

- The expensive firm's participation constraint will be binding,

  because the government wishes to keep the price p_X low

  to reduce the deadweight loss of extra taxation, the $-tp_X$ term.

√ $p_X = f(x - c_X)$

√ $\pi_N(X) > 0$

√ The normal firm's participation constraint is nonbinding.

√ (If $\pi_N(N) = 0$, then $\pi_N(X) \leq 0$,

so that $\pi_X(X) < 0$.)

- The normal firm's incentive compatibility constraint must be binding,

  because if the pair $(c_N,\ p_N)$ were strictly more attractive

  for the normal firm,

  the government could reduce the price p_N and

  save on the $-tp_N$ term.

√ $$p_N = f(-c_N) + f(x - c_X) - f(-c_X)$$

○ The government's maximization problem rewritten

√ $$\underset{c_N,\, c_X}{\textit{Maximize}} \quad \theta\,\{B - (1+t)\,c_X - t\,f(x - c_X) - f(x - c_X)\}$$

$$+ (1-\theta)\,\{B - (1+t)\,c_N - t\,f(-c_N) - t\,f(x - c_X)$$

$$+ t\,f(-c_X) - f(-c_N)\}$$

√ the expensive firm's incentive compatibility constraint

- The first-order condition with respect to c_N

  - √ $f'(\,-c_N) \;=\; 1$

  - √ $f'(e_N^{**} - \overline{c}) \;=\; 1$

  - √ The normal firm chooses the efficient effort level e^* in equilibrium: $e_N^{**} \;=\; e^*$.

  - √ $c_N^{**} \;=\; c_N^*$

  - √ $p_N^{**} \;=\; p^* \;+\; f(x - c_X^{**}) \;-\; f(\,-c_X^{**})$

- √ Incomplete information increases the price for the normal firm:

$$p_N^{**} > p^*.$$

- √ The normal firm earns informational rents –

  that is, it earns more than its reservation utility

  in the game with incomplete information.

- √ Since the expensive firm will earn exactly zero,

  this means that the government is on average providing its supplier

  with an above-market rate of return,

  because that is the way to induce normal suppliers

  to reveal that they do not have expensive problems.

- The first-order condition with respect to c_X

  - √ $f'(x - c_X) = 1 - \{(1-\theta)/\theta(1+t)\}\{t f'(x - c_X) - t f'(-c_X)\}$

  - √ $f'(x - c_X^{**}) = f'(e_X^{**} - \overline{c}) < 1$

  - √ The <u>expensive firm</u> chooses the effort level which is <u>less</u> than e^*:

    $e_X^{**} < e^*$.

  - √ $c_X^{**} = \overline{c} + x - e_X^{**} > \overline{c} + x - e^* = c_X^*$

  - √ $p_X^{**} = f(x - c_X^{**}) = f(e_X^{**} - \overline{c}) < f(e^* - \overline{c}) = p^*$

- ○ The expensive firm's incentive compatibility constraint is satisfied with a strict inequality, and is nonbinding.

  - √ $\pi_X(X) = p_X - f(x - c_X) \geq \pi_X(N) = p_N - f(x - c_N)$

  - √ $p_X^{**} - f(x - c_X^{**}) > p_N^{**} - f(x - c_N^{**})$

    - • $p_N^{**} = f(-c_N^{**}) + f(x - c_X^{**}) - f(-c_X^{**})$

    - • $f(x - c_N^{**}) - f(-c_N^{**}) > f(x - c_X^{**}) - f(-c_X^{**})$

- Summary of results

  - The government's optimal contract will induce the normal firm to exert the first-best efficient effort level and achieve the first-best cost level, but will yield that firm a positive profit.

    - √ $e_N^{**} = e^*$

    - √ $c_N^{**} = c_N^*$

    - √ $p_N^{**} > p^*$

    - √ $\pi_N(N^{**}) = p_N^{**} - f(-c_N^{**}) > 0$

- The government's <u>optimal</u> contract will induce the <u>expensive firm</u> to exert something <u>less</u> than the first-best effort level and result in a cost level <u>higher</u> than the first-best, but will yield that firm <u>zero</u> profits.

√ $e_X^{**} < e^*$

√ $c_X^{**} > c_X^*$

√ $p_X^{**} < p^*$

√ $\pi_X(X^{**}) = p_X^{**} - f(x - c_X^{**}) = 0$

- Comparison

  - √ $e_N^{**} = e^* > e_X^{**}$

  - √ $c_N^{**} = c_N^* < c_X^* < c_X^{**}$

  - √ $p_N^{**} > p^* > p_X^{**}$

  - √ $\pi_N(N^{**}) > \pi_X(X^{**}) = 0$

- There is a trade-off between the government's two objectives of

    inducing the correct amount of effort and

    minimizing the subsidy to the firm.

  - √ The government offers a contract that pays the normal firm more than under complete information

    to prevent it from mimicking the expensive firm and

    choosing an inefficiently low effort.

- √ The expensive firm chooses an inefficiently low effort, because if it were assigned greater effort it would have to be paid a greater subsidy, which would tempt the normal firm to imitate it.

- √ In equilibrium, the government has compromised by having some probability of an inefficiently high subsidy ex post, and some probability of inefficiently low effort.

- ♦ Does the optimal separating contract yield higher welfare than the cheapest pooling contract?

  - ○ The optimal separating contract

    - √ If a firm announces its type to be normal and incurs a cost of c_N^{**}, then the government pays the firm its observed cost c_N^{**} and a price of p_N^{**}.

- √ If a firm announces its type to be <u>expensive</u> and incurs a <u>cost</u> of c_X^{**}, then the government pays the firm its observed <u>cost</u> c_X^{**} and a <u>price</u> of p_X^{**}.

- √ If a firm announces its <u>type</u> and does <u>not</u> achieve the <u>cost level</u> appropriate to its announced type, then the government pays the firm its observed <u>cost</u> and a <u>price</u> of zero.

- The cheapest pooling contract

  - √ If a firm annonces its type to be expensive or normal and incurs a cost of $c_P = \overline{c} + x - e^*$, then the government pays the firm its observed cost c_P and a price of p^*.

  - √ If a firm does not achieve the cost level of $c_P = \overline{c} + x - e^*$, then the government pays the firm its observed cost and a price of zero.

♦ Procurement III: Moral Hazard with Hidden Information

- Players

  √ the government and the firm

- The order of play

  1 The government offers a contract $\{w(m) = c(m) + p(m),\ c(m)\}$ which pays the firm its observed cost c and a price p if it announces its type to be m and incurs cost $c(m)$, and pays the firm zero otherwise.

  2 The firm accepts or rejects the contract.

3 Nature assigns the firm expensive problems with the project, which add costs of x, with probability θ.

A firm is thus "normal," with type N and $s = 0$, or "expensive," with type X and $s = x$.

Only the firm observes its type.

4 If the firm accepted, it announces its type to be m and chooses effort level e, unobserved by the government.

5 If the firm accepted, it finishes the missile at a cost of $c = \overline{c} + x - e$ or $c = \overline{c} - e$, which is observed by the government, plus an additional cost of $f(e - \overline{c})$ that the government does not observe. The government reimburses $c(m)$ and pays $p(m)$.

- Payoffs
  - √ the same as in Procurement I

- The firm's type is not known to either player until after the contract is agreed upon.
  - √ The firm, however, learns its type before it must choose its effort level.

- The optimal pair of separating contracts

  - The government's maximization problem

    - $$\underset{c_N,\, c_X,\, p_N,\, p_X}{\text{Maximize}} \quad \theta \{B - (1 + t)\, c_X - t\, p_X - f(x - c_X)\}$$
      $$+ (1 - \theta) \{B - (1 + t)\, c_N - t\, p_N - f(- c_N)\}$$

- The participation constraint

  - √ $\theta \{p_X - f(x - c_X)\} + (1 - \theta)\{p_N - f(-c_N)\} \geq 0$

  - √ just one participation constraint,

    because the firm does not know its type at the time it agrees

    to the contract

  - √ The maximization problem is less constrained.

  - √ The payoff of the expensive type can be negative.

- The incentive compatibility constraints

  - ✓ for the expensive firm

$$\pi_X(X) = p_X - f(x - c_X) \geq \pi_X(N) = p_N - f(x - c_N)$$

  - ✓ for the normal firm

$$\pi_N(N) = p_N - f(-c_N) \geq \pi_N(X) = p_X - f(-c_X)$$

- ○ The <u>participation</u> constraint will be <u>binding</u>,

  because the government wants to keep the <u>deadweight loss</u>

  from taxation <u>low</u>.

√ $p_X = f(x - c_X) - \{(1 - \theta)/\theta\}\{p_N - f(- c_N)\}$

√ There is <u>no</u> informational rent ex ante.

- The normal firm's incentive compatibility constraint will be binding.

  - √ That firm does not want to admit that it can reduce cost easily, so it has a strong incentive to imitate the expensive firm, and must be bribed not to.

  - √ $p_N - f(-c_N) = p_X - f(-c_X)$

- $p_N = \theta\,\{f(x - c_X) - f(-c_X)\} + f(-c_N)$

  $p_X = (1 - \theta)\,\{f(x - c_X) - f(-c_X)\}$

○ The government's maximization problem <u>rewritten</u>

√ $\underset{c_N,\, c_X}{Maximize} \quad \theta\,[B - (1+t)\,c_X - t\,(1-\theta)\,\{f(x - c_X) - f(-c_X)\}$

$$- f(x - c_X)] + (1-\theta)\,\Big(B - (1+t)\,c_N$$

$$- t\,[\theta\,\{f(x - c_X) - f(-c_X)\} + f(-c_N)] - f(-c_N)\Big)$$

- The first-order condition with respect to c_N
  - √ $f'(-c_N) = 1$
  - √ The crucial <u>efficiency</u> condition is satisfied: $f'(e_N^{***} - \overline{c}) = 1$.
  - √ The <u>normal firm</u> chooses the <u>efficient</u> effort level e^* in equilibrium: $e_N^{***} = e^*$.
  - √ $c_N^{***} = c_N^*$
  - √ $p_N^{***} = \theta \{f(x - c_X^{***}) - f(-c_X^{***})\} + p^*$
  - √ $p_N^{***} > p^*$

- √ The normal firm earns an informational rent because partway through the game it learns its type and the government does not, and the government needs to pay something to induce it to admit to its low-cost type.

- √ The expensive firm must earn less than its reservation utility so that the overall participation constraint will be satisfied as an equality.

- The first-order condition with respect to c_X

  - √ $f'(x - c_X) = (1 + t) - 2(1 - \theta)\{t f'(x - c_X) - t f'(-c_X)\}$

    ?

- The expensive firm's <u>incentive compatibility</u> constraint is satisfied.

  - √ $p_X^{***} - f(x - c_X^{***}) > p_N^{***} - f(x - c_N^{***})$

    ?

- Summary of results

  - The government's <u>optimal</u> contract will induce the <u>normal firm</u>

    to exert the <u>first-best</u> efficient effort level and

    achieve the <u>first-best</u> cost level,

    but will yield that firm a <u>positive</u> payoff,

    though <u>smaller</u> (?) than in Procurement II.

    - √ $e_N^{***} = e^*$
    - √ $c_N^{***} = c_N^*$
    - √ $p_N^{***} > p^*$
    - √ $p_N^{***} - f(-c_N^{***}) > 0$

- The government's optimal contract will induce the expensive firm to exert something less than the first-best effort level, though more (?) than in Procurement II, and result in a cost level higher (?) than the first-best and a negative payoff.

- Overall, the firm's expected payoff will be zero.

- $e^{*} > e_{X}^{***} > e_{X}^{**}$

  $p_{N}^{**} > p_{N}^{***} > p^{*}$

  ?

- This is what one might expect of moral hazard with hidden information as compared to adverse selection.

  - The principal's maximization problem has three constraints to satisfy, instead of four, and
    this results in effort choices closer to the first-best.

  - Effort choices still do not always reach the first-best, however,
    because information about player types does become asymmetric midway through the game, and
    the contract has to be designed to induce the informed player, the firm, to disclose its information.