Chapter 11 Signalling

11.1 The Informed Player Moves First: Signalling

- Signalling is a way for an agent to communicate his type
  under adverse selection.

- The signalling contract specifies a wage
  that depends on an observable characteristic – the signal –
  which the agent chooses for himself after Nature chooses his type.

- If the agent chooses his signal before the contract is offered,
  he is signalling to the principal.

- If he chooses the signal afterwards, the principal is screening him.

- Signalling costs must differ between agent types for signalling to be useful.

- The outcome is often inefficient.

- Spence (1973) introduced the idea of signalling in the context of education.

  ✓ the notion that education has no direct effect on a person's ability to be productive in the real world
  but useful for demonstrating his ability to employers

- Education I

  - Players

    - ✓ a worker and two employers

  - The order of play

    0 Nature chooses the worker's ability $a \in \{2,\ 5.5\}$,

    the *Low* and *High* ability each having probability 0.5.

    The variable a is observed by the worker,

    but not by the employers.

1 The worker chooses education level $s \in \{0, 1\}$.

2 The employers each offer a wage contract $w(s)$.

3 The worker accepts a contract, or rejects both of them.

4 Output equals a.

- Payoffs

  - ✓ The worker's payoff is his wage minus his cost of education.

$$\pi_{worker} = w - 8\,s/a \quad \text{if the worker accepts contract } w$$

$$0 \quad \text{if he rejects both contracts}$$

  - ✓ Each employer's payoff is his profit.

$$\pi_{employer} = a - w \quad \text{for the employer whose contract is accepted}$$

$$0 \quad \text{for the other employer}$$

- Output is assumed to be a noncontractible variable and there is no uncertainty.

- The employers compete profits down to zero and the worker receives the gains from trade.

- The worker's strategy

  ✓ his education level

  ✓ his choice of employer

- The employers' strategies

  - ✓ the contracts they offer

    giving wages as functions of education level

- The key to the model is that the signal, education, is less costly

  for workers with higher ability.

  - ✓ This is what permits separation to occur.

- Pooling and Separating Equilibria

  - Pooling Equilibrium 1.1

    - ✓ $s(Low) = s(High) = 0$

      $w(0) = w(1) = 3.75$

      $Prob\,(a = Low \mid s = 1) = 0.5$

    - ✓ a perfect Bayesian equilibrium

- ✓ out-of-equilibrium behavior

- ✓ The beliefs are passive conjectures:

  The employers believe that a worker who chooses $s = 1$ is

  Low with the prior probability.

- ✓ Given this belief,

  both types of workers realize that education is useless.

- Separating Equilibrium 1.2

  - $s(Low) = 0 \qquad s(High) = 1$

    $w(0) = 2 \qquad w(1) = 5.5$

  - A pair of separating contracts must maximize the utility of the *High*s and the *Low*s subject to two sets of constraints:

    - the participation constraints that the employers can offer the contracts without making losses, and

    - the self-selection constraints

✓ the participation constraints for the employers

- $w(0) \leq a_L = 2$ and $w(1) \leq a_H = 5.5$

- Competition between the employers makes these expressions hold as equalities.

✓ the self-selection constraint of the *Low*s

- $U_L(s=0) = w(0) - 0 \geq w(1) - 8/2 = U_L(s=1)$

✓ the self-selection constraint of the *High*s

- $U_H(s=1) = w(1) - 8/5.5 \geq w(0) - 0 = U_H(s=0)$

✓ We do not need to worry about a nonpooling constraint for this game.

- The reason this does not matter is
  that the employers do not compete by offering contracts,
  but by reacting to workers who have acquired education.

- That is why this is signalling and not screening:
  the employers cannot offer contracts in advance
  that change the workers' incentives to acquire education.

✓ We can test the equilibrium by looking at the best responses.

✓ The separating equilibrium does not need to specify beliefs.

- Either of the two educaton levels might be observed in equilibrium,
  so Bayes' Rule always tells the employers how to interpret what they see.

- Another pooling equilibrium?

  - ✓ $s(Low) = s(High) = 1$

    $w(0) = ?$ $\qquad$ $w(1) = 3.75$

    $Prob\,(a = Low \mid s = 0) = ?$

  - ✓ This is <u>not</u> an equilibrium.

  - ✓ This would violate <u>incentive compatibility</u> for the *Low* workers.

    - $U_L(s = 0) = w(0) - 0 > 3.75 - 8/2 = U_L(s = 1)$

- Separation is possible because education is more costly for workers if their ability is lower.

  - ✓ This requirement of different signalling costs is the single-crossing property.

- A strong case can be made that the beliefs required for the pooling equilibria are not sensible.

  - ✓ the equilibrium refinements

- One suggestion is to inquire into whether one type of player could not possibly benefit from deviating,
  no matter how the uninformed player changed his beliefs
  as a result.

- Here, the *Low* worker could never benefit from deviating from Pooling Equilibrium 1.1.

- The more reasonable belief seems to be
  that a worker who acquires eduation is a *High*,
  which does not support the pooling equilibrium.

- If side payments are not possible,

  Separating Equilibrium 1.2 is second-best efficient

  in the sense that a social planner could not make

  both types of workers better off.

- Separation helps the high-ability workers

  even though it hurts the low-ability workers.

11.2 Variants on the Signalling Model of Education

- Education II: Modelling Trembles So Nothing Is Out of Equilibrium

  - The order of play

    0 Nature chooses the worker's ability $a \in \{2,\ 5.5\}$,

    each ability having probability 0.5.

    (a is observed by the worker, but not by the employers.)

    With probability 0.001,

    Nature endows a worker with free education of $s = 1$.

1 The worker chooses education level $s \in \{0, 1\}$.

2 The employers each offer a wage contract $w(s)$.

3 The worker accepts a contract, or rejects both of them.

4 Output equals a.

- Payoffs

  - $\pi_{worker} = w - 8\,s/a$ if the worker accepts contract w (ordinarily)

    w if he accepts contract w (with free education)

    0 if he does not accept a contract

- The advantage is that the assumptions on beliefs are put in the rules of the game along with the other assumptions.

- Education II has almost the same two equilibria as Education I, without the need to specify beliefs.

- Even that small amount of separation allows the employers to use Bayes' Rule and eliminates the need for exogenous beliefs.

- Education III: No Separating Equilibrium, Two Pooling Equilibria

  - Modify Education I by changing the possible worker abilities

    from $\{2,\ 5.5\}$ to $\{2,\ 12\}$.

  - The separating equilibrium vanishes.

    - ✓ The self-selection and zero-profit constraints cannot be satisfied simultaneously,

      because the *Low* type is willing to acquire $s = 1$

      to obtain the high wage.

- Pooling Equilibrium 3.1

  - ✓ $s(Low) = s(High) = 0$

    $w(0) = w(1) = 7$

    $Prob\,(a = Low \mid s = 1) = 0.5$

    (passive conjectures)

- Pooling Equilibrium 3.2

  - ✓ $s(Low) = s(High) = 1$

    $w(0) = 2 \qquad w(1) = 7$

    $Prob\,(a = Low \mid s = 0) = 1$

  - ✓ First-best efficiency is lost.

  - ✓ This equilibrium is not even second-best efficient.

  - ✓ The inefficiency is purely a problem of unfortunate expectations.

- ✓ The implied threat to pay a low wage to an uneducated worker never needs to be carried out,
  so the equilibrium is still called a pooling equilibrium.

- ✓ Note that perfectness does not rule out threats based on beliefs.

- ✓ The model imposes these beliefs on the employer, and
  he would carry out his threats,
  because he believes they are best responses.

- These first three games illustrate the basics of signalling:
  - ✓ Separating and pooling equilibria both may exist,
  - ✓ out-of-equilibrium beliefs matter, and
  - ✓ sometimes one perfect Bayesian equilibrium can Pareto-dominate others.

- Education IV: Continuous Signals and Continua of Equilibria

  - Players

    - ✓ a worker and two employers

  - The order of play

    0 Nature chooses the worker's ability $a \in \{2,\ 5.5\}$,

    the *Low* and *High* ability each having probability 0.5.

    The variable a is observed by the worker,

    but not by the employers.

1 The worker chooses education level $s \in [0, \infty)$.

2 The employers each offer a wage contract $w(s)$.

3 The worker accepts a contract, or rejects both of them.

4 Output equals a.

- Payoffs

  - ✓ The worker's payoff is his wage minus his cost of education.

$$\pi_{worker} = w - 8\,s/a \quad \text{if the worker accepts contract } w$$

$$0 \quad \text{if he rejects both contracts}$$

  - ✓ Each employer's payoff is his profit.

$$\pi_{employer} = a - w \quad \text{for the employer whose contract is accepted}$$

$$0 \quad \text{for the other employer}$$

- The game now has continua of pooling and separating equilibria which differ according to the value of education chosen.

- Pooling Equilibrium 4.1

  ✓ $s(Low) = s(High) = s^*$ where $s^* \in [0, \overline{s}]$

  $w(s^*) = 3.75$ $w(s \neq s^*) = 2$

  $Prob(a = Low \mid s \neq s^*) = 1$

  ✓ The critical value $\overline{s}$ can be discovered from the "incentive compatibility constraint" of the *Low* type, which is binding if $s^* = \overline{s}$.

✓ The most tempting deviation is to zero education,

so that is the deviation that appears in the constraint.

- $U_L(s=0) \;=\; 2 \;\leq\; U_L(s=s^*) \;=\; 3.75 \;-\; 8\,s^*/2$

✓ $\overline{s} \;=\; 7/16$

✓ The incentive compatibility constraint of the *High* type

is not binding.

- $U_H(s=0) \;=\; 2 \;\leq\; U_H(s=s^*) \;=\; 3.75 \;-\; 8\,s^*/5.5$

- Separating Equilibrium 4.2

✓ $s(Low) = 0 \quad s(High) = s^* \qquad$ where $s^* \in [\underline{s}, \bar{\bar{s}}]$

$w(s^*) = 5.5 \qquad w(s \neq s^*) = 2$

$Prob(a = Low \mid s \notin \{0, s^*\}) = 1$

✓ Note that there are possible out-of-equilibrium actions even in a separating equilibrium.

✓ The critical value $\bar{s}$ can be discovered from the incentive compatibility constraint of the *Low* type, which is binding if $s^* = \bar{s}$.

- $U_L(s=0) \;=\; 2 \;\geq\; U_L(s=s^*) \;=\; 5.5 \;-\; 8\,s^*/2$

✓ $\overline{s} \;=\; 7/8$

✓ If the education needed for the wage of 5.5 is too great, the *High* workers will give up on education too.

- $U_H(s=0) \;=\; 2 \;\leq\; U_H(s=s^*) \;=\; 5.5 \;-\; 8\,s^*/5.5$

✓ $\overline{\overline{s}} \;=\; 77/32$

- The big difference from Education I is that Education IV has Pareto-ranked equilibria.

  - ✓ Pooling can occur not just at zero education, but at positive levels, and the pooling equilibria with positive education levels are all Pareto inferior.

  - ✓ Also, the separating equilibria can be Pareto ranked, since separation with $s^* = \overline{s}$ dominates separation with $s^* = \overline{\overline{s}}$.

- Education IV shows how restricting the strategy space can alter the kinds of equilibria that are possible.

11.3 General Comments on Signalling in Education

- Signalling and Similar Phenomena
- Problems in Applying Signalling to Education
- Productive Signalling