25 January 2006. Eric Rasmusen, Erasmuse@indiana.edu. Http://www.rasmusen.org. Overheads for parts of Sections 6.1 and 6.3 of *Games and Information*

6 Dynamic Games with Incomplete Information

Entry Deterrence II: Fighting Is Never Profitable

In Entry Deterrence II, $X = 1$.

The sensible Nash equilibrium is ($Enter|Strong$,$Enter|Weak$, $Collude$).

A subgame cannot start at nodes E_1 or E_2.

Thus, subgame perfectness does not rule out the implausible Nash equilibrium, ($Stay\ Out|Strong$,$Stay\ Out|Weak$, $Fight$).

Also, note that Bayesian updating fails. Suppose in the implausible equilibrium that the entrant enters, deviating from equilibrium. What is the incumbent's belief, $Pr(Strong|Enter)$?

$$Pr(Strong|Enter) = \frac{Pr(Enter|Strong)Pr(Strong)}{Pr(Enter)}.$$

But $Pr(Enter) = 0$, and we can't divide by zero, even if we have a zero in the numerator too.

Perfect Bayesian Equilibrium

A **perfect bayesian equilibrium** *is a strategy profile s and a set of beliefs μ such that at each node of the game:*

(1) The strategies for the remainder of the game are Nash given the beliefs and strategies of the other players.
2) The beliefs at each information set are rational given the evidence appearing thus far in the game (meaning that they are based, if possible, on priors updated by Bayes' s Rule, given the observed actions of the other players under the hypothesis that they are in equilibrium).

In Entry Deterrence II, a PBE is

Entrant: $Enter|Weak$, $Enter|Strong$
Incumbent: $Collude$
Beliefs: $Prob(\ Strong|\ Stay\ Out) = 0.4$

There is no equilibrium with ($Stay\ Out|Strong$,$Stay\ Out|\ Weak$, $Fight$), because no out-of-equilibrium beliefs can be found to support those strategies.

Entry Deterrence III: Fighting Is Sometimes Profitable

In Entry Deterrence III, $X = 60$.

Fighting is more profitable for the incumbent than collusion if the entrant is $Weak$.

A plausible pooling equilibrium for Entry Deterrence III

Entrant: $Enter|Weak$, $Enter|Strong$
Incumbent: $Collude$, Out-of-equilibrium beliefs: $Prob(Strong|\ Stay\ Out) = 0.5$

The expected payoff to the incumbent from choosing $Fight$ is 30 ($= 0.5[0] + 0.5[60]$), which is less than the payoff of 50 from $Collude$.

The incumbent will collude, so the entrant enters. The entrant may know that the incumbent's payoff is actually 60, but that is irrelevant to the incumbent's behavior.

The out-of-equilibrium belief does not matter to this first equilibrium.

An implausible equilibrium for Entry Deterrence III

Entrant: $Stay\ Out|Weak$, $Stay\ Out|Strong$
Incumbent: $Fight$,
Out-of-equilibrium beliefs: $Prob(Strong|Enter) = 0.1$

If the entrant were to deviate and enter, the incumbent would calculate his payoff from fighting to be 54 ($= 0.1[0] + 0.9[60]$), more than the *Collude* payoff of 50. The entrant would therefore stay out.

A **separating** equilibrium would look like this:

A conjectured separating equilibrium for Entry Deterrence III
Entrant: *Stay Out*|*Weak*, *Enter*|*Strong*
Incumbent: *Collude*

No out-of-equilibrium beliefs are specified for the conjectures in the separating equilibrium because there is no out-of-equilibrium behavior about which to specify them.

Entry Deterrence IV: The Incumbent Benefits from Ignorance

Let $X = 300$, so fighting is even more profitable than in Entry Deterrence III but the game is otherwise the same.

The unique (in its strategies) perfect bayesian equilibrium in pure strategies is

Equilibrium for Entry Deterrence IV
Entrant: $Stay\ Out\,|Weak$, $Stay\ Out\,|Strong$
Incumbent: $Fight$,
Out-of-equilibrium beliefs: $Prob(Strong|Enter) = 0.5$ (passive conjectures)

Entry Deterrence V: Lack of Common Knowledge of Ignorance

In Entry Deterrence V, it may happen that both the entrant and the incumbent know the payoff from (*Enter*, *Fight*), but the entrant does not know whether the incumbent knows. The information is known to both players, but is not common knowledge.

The game begins with Nature assigning the entrant a type, *Strong* or *Weak*, observed by the entrant but not by the incumbent.

Next, Nature moves again and either tells the incumbent the entrant's type or remains silent, observed by the incumbent, but not by the entrant.

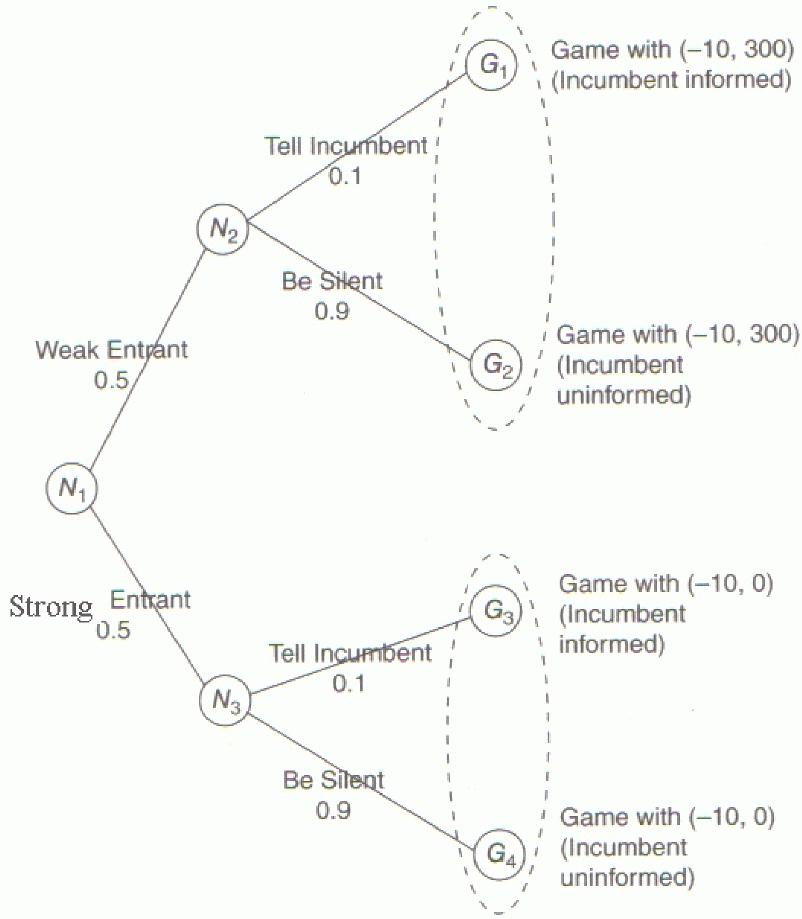

Figure 6.3 Entry Deterrence V

Equilibrium for Entry Deterrence V

Entrant: *Stay Out*|*Weak*, *Stay Out*|*Strong*
Incumbent: *Fight*|*Nature said "Weak"*, *Collude* |*Nature said "Strong"*, *Fight* |*Nature said nothing*, Out-of- equilibrium beliefs: *Prob*(*Strong*|*Enter, Nature said nothing)* = 0.5 (passive conjectures)

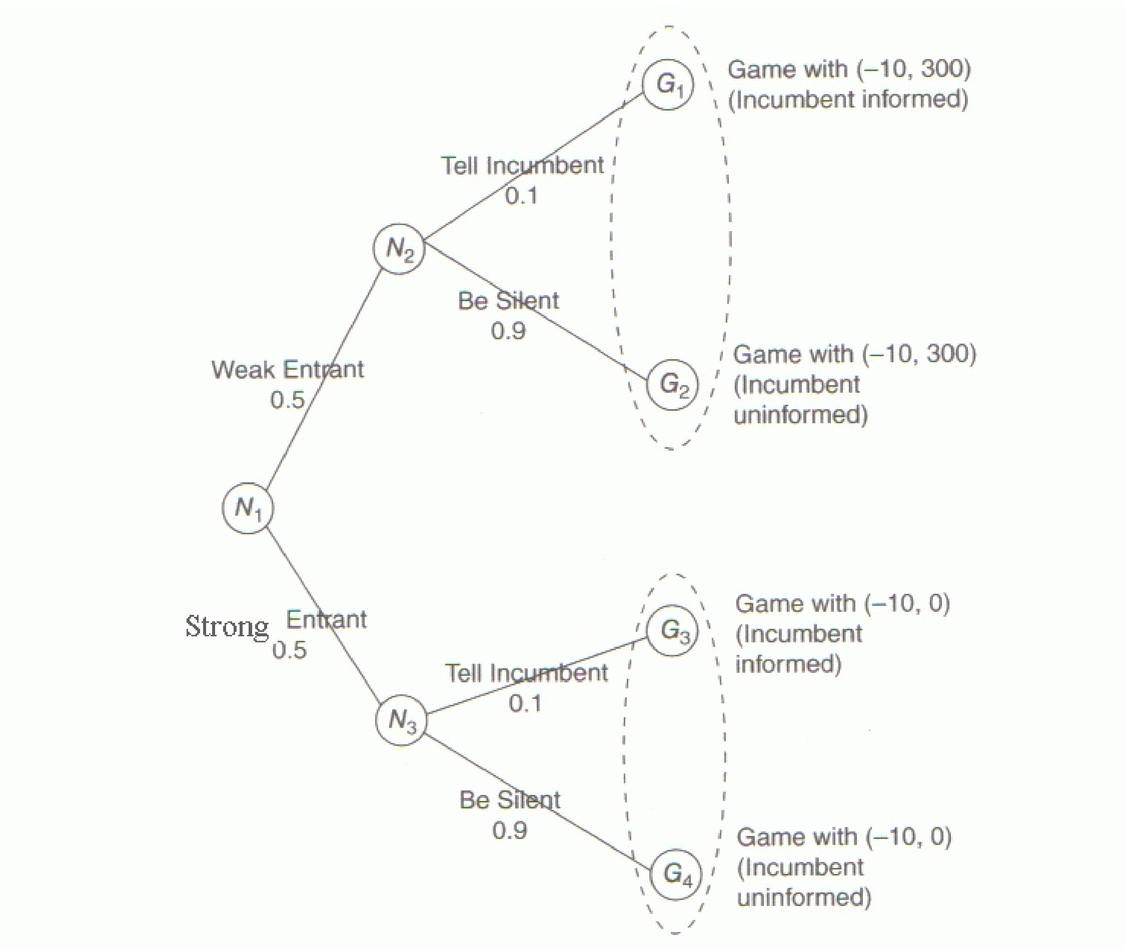

Figure 6.3 Entry Deterrence V

The incumbent will fight for either of two reasons.

With probability 0.9, Nature has said nothing and the incumbent calculates his expected payoff from *Fight* to be 150.

With probability 0.05 (= 0.1[0.5]) Nature has told the incumbent that the entrant is weak and the payoff from *Fight* is 300.

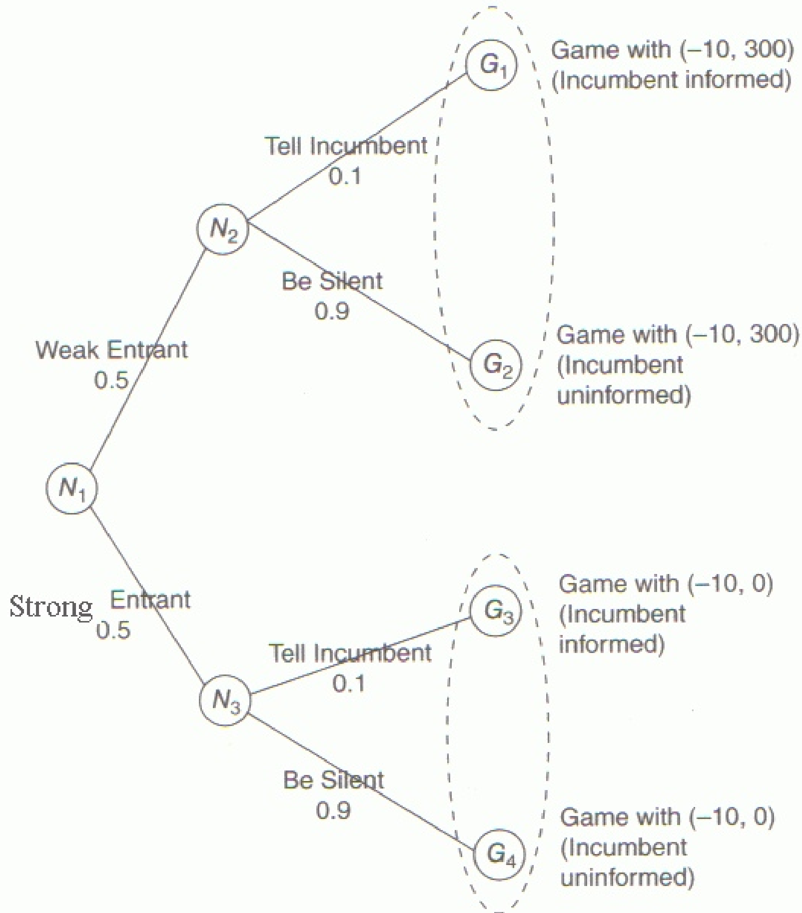

Figure 6.3 Entry Deterrence V

Even if the entrant is strong and Nature tells this to the incumbent, the entrant would choose *Stay Out*, because he does not know that the incumbent knows, and his expected payoff from *Enter* would be -5 $(= [0.9][-10] + 0.1[40])$.

If it were common knowledge that the entrant was strong, the entrant would enter and the incumbent would collude. If it is known by both players, but not common knowledge, the entrant stays out.