8 Further Topics in Moral Hazard

This is designed for one 75-minute lecture using *Games and Information*. Probably I have more material than I will end up covering.

These slides just cover sections 8.1 (efficiency wage), 8.6 (teams) and 8.7 (multi- tasking).

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8.1 Efficiency Wages

Is the aim of an incentive contract to punish the agent if he chooses the wrong action?

Not exactly.

Rather, it is to create a difference between the agent's expected payoff from right and wrong actions.

That can be done either with the stick of punishment or the carrot of reward.

The Lucky Executive Game

Players: A corporation and an executive.

The Order of play

1 The corporation offers the executive a contract which pays $w(q) \geq 0$ depending on profit, q.

2 The executive accepts the contract, or rejects it and receives his reservation utility of $\overline{U} = 5$

3 The executive exerts effort e of either 0 or 10.

4 Nature chooses profit according to Table 1.

Payoffs: Both players are risk neutral. The corporation's payoff is $q - w$. The executive's payoff is $(w - e)$ if he accepts the contract.

Table 1: Output in the Lucky Executive Game

	Probability of Outputs		
Effort	0	400	Total
Low $(e = 0)$	0.5	0.5	1
High $(e = 10)$	0.1	0.9	1

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Since both players are risk neutral, you might think that the first-best can be achieved by selling the store, putting the entire risk on the agent. The participation constraint if the executive exerts high effort is

$$0.1[w(0) - 10] + 0.9[w(400) - 10] \geq 5, \tag{1}$$

so his expected wage must equal 15.

The incentive compatility constraint is

$$0.5w(0) + 0.5w(400) \leq 0.1w(0) + 0.9w(400) - 10, \tag{2}$$

which can be rewritten as $w(400) - w(0) \geq 25$, so the gap between the executive's wage for high output and low output must equal at least 25.

A contract that satisfies both constraints is $\{w(0) = -345, w(400) = 55\}$.

But this contract is not feasible, because the game requires $w(q) \geq 0$: the **bankruptcy constraint**.

The participation constraint if the executive exerts high effort is

$$0.1[w(0) - 10] + 0.9[w(400) - 10] \geq 5, \tag{3}$$

so his expected wage must equal 15.

The incentive compatility constraint is

$$0.5w(0) + 0.5w(400) \leq 0.1w(0) + 0.9w(400) - 10, \tag{4}$$

What can be done is to use the carrot instead of the stick and abandon satisfying the participation constraint as an equality.

All that is needed for constraint (4) is a gap of 25 between the high wage and the low wage.

Setting the low wage as low as is feasible, the corporation can use the contract $\{w(0) = 0, w(400) = 25\}$ and induce high effort.

The executive's expected utility, however, will be $0.1(0)+0.9(25)-10 = 12.5$, more than double his reservation utility of 5.

He is very happy in this equilibrium– but the corporation is reasonably happy, too. The corporation's payoff is $337.5(= 0.1(0-0)+0.9(400-25)$, compared with the $195(= 0.5(0-5)+0.5(400-5))$ it would get if it paid a lower expected wage.

This discussion should remind you of Section 5.4's Product Quality Game.

There too, purchasers paid more than the reservation price in order to give the seller an incentive to behave properly, because a seller who misbehaved could be punished by termination of the relationship.

The key characteristics of such models are a constraint on the amount of contractual punishment for misbehavior and a participation constraint that is not binding in equilibrium.

Repetition allows for a situation in which the agent could considerably increase his payoff in one period by misbehavior such as stealing or low quality but refrains because he would lose his position and lose all the future efficiency wage payments.

*8.6 Joint Production by Many Agents: The Holmstrom Teams Model

A **team** *is a group of agents who independently choose effort levels that result in a single output for the entire group.*

Teams

(Holmstrom [1982])

Players

A principal and n agents.

The order of play

1 The principal offers a contract to each agent i of the form $w_i(q)$, where q is total output.
2 The agents decide whether or not to accept the contract.
3 The agents simultaneously pick effort levels e_i, $(i = 1, \ldots, n)$.
4 Output is $q(e_1, \ldots e_n)$.

Payoffs

If any agent rejects the contract, all payoffs equal zero. Otherwise,

$$\pi_{principal} = q - \sum_{i=1}^{n} w_i;$$

$$\pi_i = w_i - v_i(e_i), \text{ where } v_i' > 0 \text{ and } v_i'' > 0.$$

Despite the risk neutrality of the agents, "selling the store" fails to work here, because the team of agents still has the same problem as the employer had. The team's problem is cooperation between agents, and the principal is peripheral.

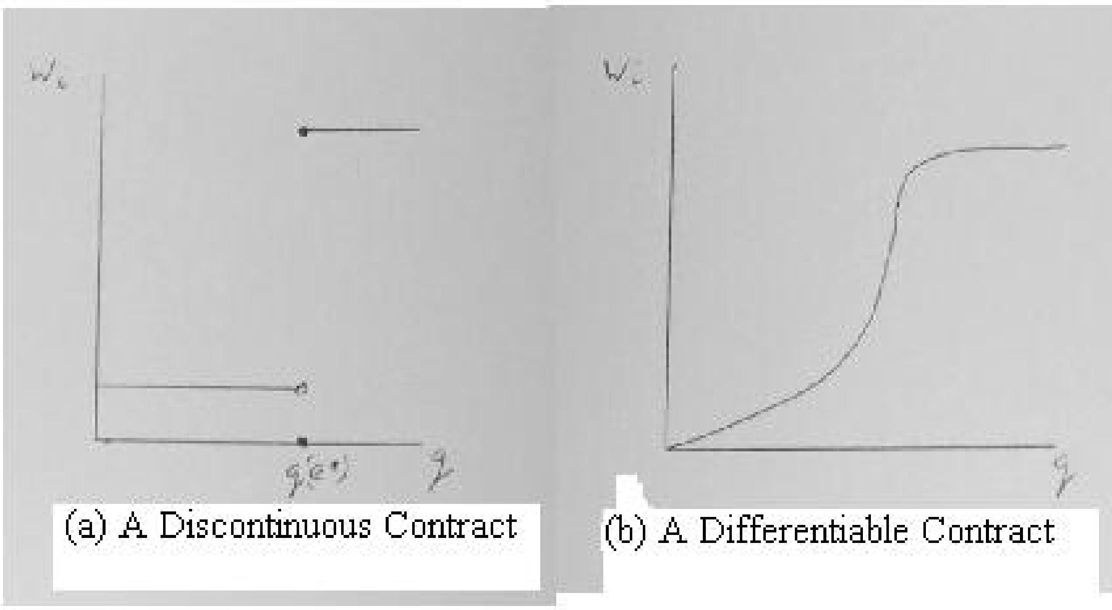

Figure 4: Contracts in the Holmstrom Teams Model

Denote the efficient vector of actions by e^*. An efficient contract, illustrated in Figure 4(a), is

$$w_i(q) = \begin{cases} b_i \text{ if } q \geq q(e^*) \\ \\ 0 \text{ if } q < q(e^*) \end{cases} \tag{5}$$

where $\sum_{i=1}^{n} b_i = q(e^*)$ and $b_i > v_i(e_i^*)$.

Contract (5) gives agent i the wage b_i if all agents pick the efficient effort, and nothing if any of them shirks.

Proposition 1. *If there is a budget-balancing constraint, no differentiable wage contract $w_i(q)$ generates an efficient Nash equilibrium.*

Agent i's problem is

$$\underset{e_i}{Maximize} \quad w_i(q(e)) - v_i(e_i). \tag{6}$$

His first-order condition is

$$\left(\frac{dw_i}{dq}\right)\left(\frac{dq}{de_i}\right) - \frac{dv_i}{de_i} = 0. \tag{7}$$

With budget balancing and a linear utility function, the pareto optimum maximizes the sum of utilities (something not generally true), so the optimum solves

$$\underset{e_1,\ldots,e_n}{Maximize} \;\; q(e) - \sum_{i=1}^{n} v_i(e_i) \tag{8}$$

The first-order condition is that the marginal dollar contribution to output equal the marginal disutility of effort:

$$\frac{dq}{de_i} - \frac{dv_i}{de_i} = 0. \tag{9}$$

Equation (9) contradicts equation (7), the agent's first-order condition, because $\frac{dw_i}{dq}$ is not equal to one.

*8.7 The Multitask Agency Problem : Multitasking I: Two Tasks, No Leisure Holmstrom & Milgrom (1991)

The Order of Play

1 The principal offers the agent either an incentive contract of the form $w(q_1)$ or a monitoring contract that pays m under which he pays the agent a base wage of $\overline{m}$ plus m_1 if he observes him working on Task 1 and m_2 if he observes him working on Task 2 (the $\overline{m}$ base is superfluous notation in Multitasking I, but is used in Multitasking II).
2 The agent decides whether or not to accept the contract.
3 The agent picks efforts e_1 and e_2 for the two tasks such that $e_1 + e_2 = 1$, where 1 denotes the total time available.
4 Outputs are $q_1(e_1)$ and $q_2(e_2)$, where $\frac{dq_1}{de_1} > 0$ and $\frac{dq_2}{de_2} > 0$ but we do not require decreasing returns to effort.

Payoffs: If any agent rejects the contract, all payoffs equal zero. Otherwise,

$$\begin{aligned} \pi_{principal} &= q_1 + \beta q_2 - m - w - C; \\ \pi_{agent} &= m + w - e_1^2 - e_2^2, \end{aligned} \tag{10}$$

where C, the cost of monitoring, is $\overline{C}$ if a monitoring contract is used and zero otherwise.

The first best can be found by choosing e_1 and e_2 (subject to $e_1 + e_2 = 1$) and C to maximize the sum of the payoffs,

$$\pi_{principal} + \pi_{agent} = q_1(e_1) + \beta q_2(e_2) - C - e_1^2 - e_2^2, \qquad (11)$$

In the first-best, $C = 0$ of course– no costly monitoring is needed.

Substituting $e_2 = 1 - e_1$ and using the first-order condition for e_1 yields

$$C^* = 0 \qquad e_1^* = \frac{1}{2} + \left(\frac{\frac{dq_1}{de_1} - \beta \left(\frac{dq_2}{de_2} \right)}{4} \right) \qquad (12)$$

$$e_2^* = \frac{1}{2} - \left(\frac{\frac{dq_1}{de_1} - \beta \left(\frac{dq_2}{de_2} \right)}{4} \right). \qquad (13)$$

Thus, which effort should be bigger depends on β (a measure of the relative value of Task 2) and the diminishing returns to effort in each task.

If, for example, $\beta > 1$ so Task 2's output is more valuable and the functions $q_1(e_1)$ and $q_2(e_2)$ produce the same output for the same effort, then from (13) we can see that $e_1^* < e_2^*$, as one would expect.

Can an incentive contract achieve the first best?

Define q_1^*, q_2^*, e_1^* and e_2^* as the first-best levels of those variables and define the minimum wage payment that would induce the agent to accept a contract requiring the first-best effort as

$$w^* \equiv (e_1^*)^2 + (e_2^*)^2 \tag{14}$$

What happens with the profit-maximizing flat-wage contract, which could be either the incentive contract $w(q_1) = w^*$ or the monitoring contract $\{w^*, w^*\}$? The agent's effort choice would be to split his effort equally between the two tasks, so $e_1 = e_2 = 0.5$. To satisfy the participation constraint it would be necessary that $\pi_{agent} = w^* + w - e_1^2 - e_2^2 \geq 0$, so $\pi_{agent} = w^* - 0.25 - 0.25 = 0$ and $w^* = 0.5$.

What about a sharing-rule incentive contract, in which the wage rises with output (that is, $\frac{dw}{dq_1} > 0$)?

The principal must worry about an externality of sorts: the greater the agent's effort on Task 1, the less will be his effort on Task 2. Even if extra e_1 were free, the principal might not want it– and might be willing to pay to stop it.

Consider the simplest sharing-rule contract, the linear one with $\frac{dw}{dq_1} = b$, so $w(q_1) = a + bq_1$. The agent will pick e_1 and e_2 to maximize

$$\pi_{agent} = a + bq_1(e_1) - e_1^2 - e_2^2, \tag{15}$$

subject to $e_1 + e_2 = 1$ (which allows us to rewrite the maximand in terms of just e_1, since $e_2 = 1 - e_1$). The first-order condition is

$$\frac{d\pi_{agent}}{de_1} = b\left(\frac{dq_1}{de_1}\right) - 2e_1^* - 2(1 - e_1^*)(-1) = 0, \tag{16}$$

so

$$e_1^* = \frac{1}{2} + \left(\frac{b}{4}\right)\left(\frac{dq_1}{de_1}\right). \tag{17}$$

If $e_1^* \geq 0.5$, the linear contract will work just fine.

If $e_1^* < 0.5$, the linear contract cannot achieve the first best with a positive value for b. Even under a flat wage ($b = 0$), the agent will choose $e_1 = 0.5$, which is too high.

If the principal rewards the agent for more of the observable output q_1, the principal will get too little of the unobservable output q_2. Instead, the contract must actually punish the agent for high output!

It must have at least a slightly negative value for b, so as to defeat the agent's preferred allocation of effort evenly across the tasks.

Chapter 7 compared three contracts: linear, threshold, and forcing contracts. The threshold contract will work as well or better than the linear contract in Multitasking I. It at least does not provide incentive to go above the threshold, which is positively bad in this model.

The forcing contract is even better, because the principal positively dislikes having e_1 be too great.

Thus, in equilibrium the principal chooses some contract that elicits the first-best effort e^*, such as the forcing contract,

$$\begin{aligned} w(q_1 = q_1^*) &= w^*, \\ w(q_1 \neq q_1^*) &= 0. \end{aligned} \tag{18}$$

A monitoring contract, which would incur monitoring cost $\overline{C}$, is suboptimal, since an incentive contract can achieve the first-best anyway, but let's see how the optimal monitoring contract would work.

Let us set $\overline{m} = 0$ in Multitasking I.

The agent will choose his effort to maximize

$$\begin{aligned} \pi_{agent} &= e_1 m_1 + e_2 m_2 - e_1^2 - e_2^2 \\ &= e_1 m_1 + (1-e_1) m_2 - e_1^2 - (1-e_1)^2, \end{aligned} \tag{19}$$

since with probability e_1 the monitoring finds him working on Task 1 and with probability e_2 it finds him on Task 2. Thus,

$$\frac{d\pi_{agent}}{de_1} = m_1 - m_2 - 2e_1 - 2(-1)(1-e_1) = 0 \tag{20}$$

so if the principal wants the agent to pick the particular effort $e_1 = e_1^*$ that we found in equation (13) he should choose m_1^* and m_2^* so that

$$m_1^* = 4e_1^* + m_2^* - 2 \tag{21}$$

If $e_1^* > e_2^*$, which means that $e_1^* > 0.5$, equation (21) tells us that $m_1^* > m_2^*$, just as we would expect.

We have one equation for the two unknowns of m_1^* and m_2^* in (21), so we need to add some information. Let us use the fact that if the participation constraint is satisfied exactly then we can set the agent's payoff from (19) equal to zero, which is a second equation for our two unknowns.

After going through the algebra to solve (21) together with the binding participation constraint, we get

$$m_1^* = 4e_1^* - 2(e_1^*)^2 - 1 \tag{22}$$

$$\begin{aligned} m_2^* &= [4e_1^* - 2(e_1^*)^2 - 1] + 2 - 4e_1^* \\ &= 1 - 2(e_1^*)^2 \end{aligned} \tag{23}$$

These have the expected property that $\frac{dm_1^*}{de_1^*} = -4e_1^* + 4 > 0$ and $\frac{dm_2^*}{de_1^*} = -4e_1^* < 0$.

Multitasking II: Two Tasks Plus Leisure

This game is the same as Multitasking I, except that now the agent's effort budget constraint is not $e_1 + e_2 = 1$, but $e_1 + e_2 \leq 1$.

Again let us begin with the first best. This can be found by choosing e_1 and e_2 and C to maximize the sum of the payoffs:

$$q_1(e_1) + \beta q_2(e_2) - C - e_1^2 - e_2^2, \tag{24}$$

subject to $e_1 + e_2 \leq 1$, the only change in the optimization problem from Multitasking I.

We now cannot use the trick of substituting for e_2 using the constraint $e_2 = 1 - e_1$, since it might happen that the effort budget constraint is not binding at the optimum.

Maybe $e_1^* + e_2^* = 1$, as in Multitasking I, so that the first-best effort levels are the same as in that game.

But positive leisure for the agent in the first-best, i.e., the effort budget constraint being non-binding, is a realistic case.

In Multitasking I, a flat wage led to $e_1 = e_2 = 0.5$.

In Multitasking II, it would lead to $e_1 = e_2 = 0$, quite a different result.

A low-powered incentive contract is disastrous, because pulling the agent away from high effort on Task 1 does not leave him working harder on Task 2.

A high-powered sharing-rule incentive contract in which the wage rises with output performs much better, even though we cannot reach the first best as we did in Multitasking I.

Since the flat wage leads to $e_2 = 0$ anyway, adding incentives for the agent to increase e_1 cannot do any harm.

Effort on Task 2 will remain zero– so the first-best is unreachable– but a suitable sharing rule can lead to $e_1 = e_1^*$.

The combination $(e_1 = e_1^*, e_2 = 0)$ is the second-best incentive-contract solution in Multitasking II, since at e_1^* the marginal disutility of effort equals the marginal utility of the marginal product of effort.

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That conclusion might be misleading, though. We have assumed that the disutility of effort on Task I is separable from the disutility of effort on Task II.

That is why even if the agent is devoting no effort to Task II he should not work any harder on Task I.

More realistically, the disutility of effort would be some non-separable function $f(e_1, e_2)$ such that the efforts are "substitute bads" and $\frac{d^2 f}{de_1 de_2} > 0$.

In that case, in the second-best the principal, unable to induce e_2 to be positive, would push e_1 above the first-best level, since the agent's marginal disutility of e_1 would be less at $(e_1^*, 0)$ than at (e_1^*, e_2^*).

Thus, one lesson of Multitasking II is that if an agent has a strong temptation to spend his time on tasks which have no benefit for the principal, the situation is much closer to the conventional agency models than to Multitasking I.

The first-best effort levels *can* be attained, but it requires a monitoring contract .

The agent will choose his effort to maximize

$$\pi_{agent} = \overline{m} + e_1 m_1 + e_2 m_2 - e_1^2 - e_2^2, \tag{25}$$

subject to $e_1 + e_2 \leq 1$.

Unlike in Multitasking I, the base wage $\overline{m}$ matters, since it may happen that the principal monitors the agent and finds him working on neither Task 1 nor Task 2. The base wage may even be negative, which can be interpreted as a bond for good effort posted by the agent or as a fee he pays for the privilege of filling the job and possibly earning m_1 or m_2.

The principal will pick m_1 and m_2 to induce the agent to choose e_1^* and e_2^*, so he will pick them to solve the first-order conditions of the agent's problem for e_1^* and e_2^*:

$$\begin{aligned} \frac{\partial \pi_{agent}}{\partial e_1} &= m_1 - 2e_1 = 0 \\ \frac{\partial \pi_{agent}}{\partial e_2} &= m_2 - 2e_2 = 0 \end{aligned} \tag{26}$$

These can be solved to yield $m_1 = \frac{e_1^*}{2}$ and $m_2 = \frac{e_2^*}{2}$.

We still need to determine the base wage, $\overline{m}$. Substituting into the participation constraint, which will be binding, and recalling that we defined the agent's reservation expected wage as $w^* = e_1^2 + e_2^2$,

$$\begin{aligned}
\pi_{agent} &= \overline{m} + e_1 m_1 + e_2 m_2 - e_1^2 - e_2^2 = 0 \\
&= \overline{m} + e_1^* \left(\frac{e_1^*}{2}\right) + e_2^* \left(\frac{e_2^*}{2}\right) - w^* = 0 \qquad (27) \\
&= \overline{m} + \left(\tfrac{1}{2}\right) w^* - w^* = 0
\end{aligned}$$

so $\overline{m} = \frac{w^*}{2}$.

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The base wage is thus positive; even if the principal finds the agent shirking when he monitors, he will pay him more than zero.

That is intuitive when $e_1^* + e_2^* < 1$, because then the principal wants the agent to take some leisure in equilibrium, rather than have to pay him more for a leisureless job.

It is more surprising that the base wage is positive when $e_1^* + e_2^* = 1$; that is, when efficiency requires zero leisure. Why pay the agent anything at all for inefficient behavior?

The answer is that the base wage is important only for inducing the agent to take the job and has no influence whatsoever on the agent's choice of effort.

Increasing the base wage does not make the agent more likely to take leisure, because he gets the base wage regardless of how much time he spends on each activity.

If $e_1^* + e_2^* = 1$, then the agent chooses zero leisure despite knowing that he would still receive his base pay for doing nothing, because the incentive of m_1 and m_2 is great enough that he does not want to waste any opportunity to get that incentive pay.