G751, Spring 2012: Test 2

There are 100 points in total. Please start each of the 3 questions on a new sheet of paper. This is a closed-book exam.

Question 1 (30 points) Firm Apex is aggressive, normal, or passive with probabilities .5, .3, and .2. Its rival Brydox does not know Apex's type. Set up the algebraic expressions to answer the following questions—you do not have to do the arithmetic calculations.

(a) In equilibrium, Apex introduces a new product in the first period with probability .7 if it is aggressive, .5 if it is normal, and .1 if it is passive. If Apex does introduce a new product, what does Brydox think is the probability that Apex is normal?

Use Bayes's Rule:

$$Prob(normal|introduce) = \frac{(.5)(.3)}{(.5)(.3) + (.7)(.5) + (.1)(.2)} = \frac{15}{52}.$$

(b) In the second period, if Apex has introduced one new product already, it introduces a second new product with probability .2 if it is aggressive, .1 if it is normal, and 0 if it is passive. If Apex introduces a new product in both the first and second period, what does Brydox think is the probability that Apex is aggressive?

Use Bayes's Rule twice:

$$Prob(aggressive|introduce\ once) = \frac{(.7)(.5)}{(.5)(.3) + (.7)(.5) + (.1)(.2)} = 35/52$$

35/52 is the new prior.

$$Prob(aggressive|introduce\ twice) = \frac{(.2)(35/52)}{.2(35/52) + .1(15/52)} = 70/85 = 14/17$$

(c) If Apex introduces a product in the first period but not the second, what does Brydox think at the end of the second period is the probability Apex is passive?

$$Prob(passive|introduce\ once) = \frac{(.1)(.2)}{(.5)(.3) + (.7)(.5) + (.1)(.2)} = 2/52$$

$$Prob(passive|not\ introduce\ twice) = \frac{(1)(2/52)}{.8(35/52) + .9(15/52) + (1)(2/52)} \approx .46$$

An alternative way to do the calculation is

$$Prob(passive|not\ introduce\ twice) = \frac{(1)(.2)(.1))}{.8(.5)(.7) + .9(.3(.5) + (1)(.2)(.1)} \approx .46$$

Question 2 (40 points) Firms Apex and Brydox are threatened by a new government regulations that will cost them amounts x_a and x_b if it passes. If at least one firm lobbies against the regulation at cost$C$ the regulation will fail to be implemented, with $0 < C$ and C less than either x_a or x_b. In the following year, they are threatened by another regulation costing the same x_a, x_b, and C. In the second year, however, they both must lobby or the regulation cannot be stopped. Use the notation Lobby if a firm lobbies and Don't if it doesn't.

(a) Find a perfect equilibrium in which there is no lobbying in period 2.

Remember to give the entire strategy profile, not just the actions in the second period— the question is how those actions can be part of an equilibrium. Here is an equilibrium: Apex: $(Don't, Don't|Lobby, Don't|Don't)$. Brydox: $(Lobby, Don't|Lobby, Don't|Don't$ $)$

(b) Find a perfect equilibrium in which in period 2 mixed strategies are used. In period 2, let Apex choose *Lobby* with probability θ and let Brydox choose *Lobby* with probability γ.

In Period 1, let Apex choose Don't and Brydox choose Lobby. In period 2, if Apex is indifferent between strategies then

$$\pi_a(Lobby) = -C - (1-\gamma)x_a = \pi_a(Don't) = -x_a$$

so $C + x_a - \gamma x_a = x_a$ *and* $\gamma = \frac{C}{x_a}$. *If Brydox is indifferent between strategies then*

$$\pi_b(Lobby) = -C - (1-\theta)x_b = \pi_b(Don't) = -x_b$$

so $\theta = \frac{C}{x_b}$.

(c) Find a perfect equilibrium in which there is no lobbying in period 1.

Apex: $(Don't, Don't|Lobby, Lobby|Don't)$. *Brydox:* $(Don't, Don't|Lobby, Lobby|Don't)$

Apex's equilibrium payoff is $-x_a - C$. *Deviating to Don't keeps this the same, but with the* C *incurred in the first period and the* x_a *in the second. Brydox's payoffs are analogous.*

(d) Find a perfect equilibrium in which both firms lobby in period 1. What restrictions on the parameter values are required for this equilibrium to exist?

Apex:$(Lobby, Lobby|Lobby, Don't|Don't)$, *Brydox:* $(Lobby, Lobby|Lobby, Don't|Don't)$

Apex's equilibrium payoff is $-2C$. *Deviating to Don't in the first and periods it becomes* $-x_a$. *Brydox's payoffs are analogous. This equilibrium requires* $-2C < -x_a$ *and* $-2C < -x_b$.

Question 3 (30 points). Apex is suing Brydox for patent infringement. Apex knows its probability of winning in court, which is 1 if it is strong and 0 if it is weak, but Brydox does not. Brydox thinks the probability of Apex being strong is θ. Apex must pay amount C to file suit, and then can make a take-it-or-leave-it offer to Brydox of s. If Brydox rejects the offer, Apex chooses whether or not to go to trial. If they do go to trial, Apex pays p in legal fees and Brydox pays d. If Apex wins at trial, Brydox pays Apex damages of v.

(a) What is an equilibrium in which settlement occurs?

Working backwards, Apex will go to trial if it is strong and drop the case if it is weak. Thus, Brydox thinks its payoff if it refuses the settlement is $\theta(-v-d)$. *Apex chooses the settlement amount to make accepting the settlement yield Brydox an equal payoff, so* $-s = \theta(-v-d)$ *and* $s = \theta(v+d)$. *Out-of-equilibrium beliefs do not matter. Thus, in the most desirable equilibrium, Apex files suit, offers* $s = \theta(v+d)$, *and goes to court only if it is strong; Brydox accepts any settlement of* $s \leq \theta(v+d)$ *and rejects* *any larger s, and if he rejects the settlement then he goes to trial if he is strong but not if he is weak.*

(b) What restrictions must be put on the value of c for that to be an equilibrium?

It is necessary that $c \leq \theta(v+d)$. It is also necessary that $v < p$, but that requirement isn't special to this particular equilibrium– it's necessary if Apex is ever to choose to go to trial.

(c) What is a perfect Bayesian equilibrium in which Apex sues but obtains a lower payoff than in (a)?

The key to this part of the question is to realize that if Apex choosea a particular value for s in equilibrium, then if it chooses a different value for s, that is out-of-equilibrium behavior and Brydox has to have some sort of out-of-equilibrium belief specified.

Consider the following equilibrium:

Apex files suit, offers $s = s'$, and goes to court only if it is strong; Brydox accepts any settlement of $s \leq s'$ and rejects any larger s. If he rejects the settlement then he goes to trial if he is strong but not if he is weak. Out-of-equilibrium belief: If $s > s'$, then Brydox believes that Apex is weak with probability 1.

For this to be an equilibrium, s' must be large enough that the strong Apex does not prefer going to trial. Thus, we need $s' \geq v - p$. Also, it is necessary that $s' \geq -c$ or Apex will not file suit.