The Rubinstein Bargaining Model with Both Discounting and Fixed Per-Period Costs February 12, 2020

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Abstract

Rubinstein (1982) describes two bargaining models which reach opposite conclusions. Model I, now the canonical model of bargaining, has positive discount rates and the split is close to 50-50 when discount rates are small and almost equal. Model II has a fixed cost for each period of bargaining, and the split is 100-0 when bargaining costs are small and almost equal. Rubinstein does not say what happens in a model with both discounting and bargaining costs. If the mixed model were to behave more like Model II, the Rubinstein model would be a poor fit to reality. In fact, the mixed model behaves more like Model I, so the canonical model is safe from this criticism.

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1. Introduction

Rubinstein's 1982 *Econometrica* article on bargaining has been exceptionally influential, receiving 2,003 cites in Google Scholar as of June 2008. It is a simple and elegant model of bargaining in which agreement is reached immediately, the split is close to 50-50 between identical players, and the split diverges from 50-50 depending on a simple measure of bargaining power— the discount rate. The model is useful not only in direct application, but as a component of more complex models of interactions in which players split a surplus at some point in the game they are playing. Rather than simply assume that the bargaining split is 50-50, or is $\lambda, (1-\lambda)$ for some reduced-form bargaining parameter, the modeller can build a model from primitive assumptions about payoffs and actions.

A complete reading of Rubinstein (1982), however, raises a nagging doubt. Rubinstein (1982) actually contains two simple models of bargaining, and they reach opposite conclusions. In Model I, the much-cited model based on time discounting, the split is about 50-50 when discount rates are small and almost equal. In Model II, which has a fixed cost for each period of bargaining, the split is 100-0 when bargaining costs are small and almost– but not quite– equal. Rubinstein does not say what happens in a model with both discounting and per-offer bargaining costs. If that model were to behave more like Model II, the Rubinstein model would be a poor fit to reality. We would wonder whether Model I might really be a special case that we have adopted because we like to think we are using structural models but which really is no advance over simply assuming that bargainers split the surplus 50-50. So let's take a look at the model that combines both discounting and per-offer bargaining costs.

2. The Model

Players 1 and 2 are splitting a pie of size 1. We will use x_t to denote an offer in period t that gives share x_t to player 1. Player 1 starts with an offer of a split of x_1 for himself and $(1-x_1)$ for player 2. If player 2 accepts, the game is over and the payoffs are

$$\pi_1 = x_1 - c_1 \qquad\qquad \pi_2 = 1 - x_1 - c_2. \tag{1}$$

If player 2 rejects, a period of time passes, at the end of which he pays c_2 and player 1 pays c_1. He then makes an offer of a split of x_2 for player 1 and $1-x_2$ for himself. If player 1 accepts, the game is over and the payoffs are (viewed from the start of the game)

$$\pi_1 = -c_1 + \delta_1[-c_1 + x_2] \qquad\qquad \pi_2 = -c_2 + \delta_2[-c_2 + (1-x_2)] \tag{2}$$

because payoffs are discounted by discount factors of $\delta_1 = \frac{1}{1+\rho_1} < 1$ and $\delta_2 = \frac{1}{1+\rho_2} < 1$ each period. Our main interest is in what happens when the periods are short, so the discount rates approach $\rho_1 = \rho_2 = 0$ and the discount factors approach $\delta_1 = \delta_2 = 1$.

If player 1 rejects rejects player 2's offer, another period of time elapses, the players incur costs c_1 and c_2 again, and player 1 makes the next offer, x_3. The two players make alternating offers until agreement is reached, or forever if agreement is not reached.

Rubinstein's Theorem established existence, though not uniqueness, for a more general version of payoffs that includes this particular case. He does show uniqueness and the particular equilibrium share for the two cases of just discounting and just fixed costs.

3. The Equilibrium

In any subgame perfect equilibrium, player 1's first offer will be accepted immediately. The only reason to wait is to get a bigger share $x_1 = x_1'$ later. Player 1 would prefer to offer $x_1' - c_1 + \epsilon$ now.

It follows that the equilibrium is stationary. The players' payoffs cannot get bigger or smaller, or they would make more generous offers now that would equal any bigger payoffs without having to pay the bargaining costs. So let us proceed assuming that player 1 always chooses the same x_1 and player 2 always chooses the same x_2.

Let Player 1 take x_2, the share player 2 will offer player 1, as given. Player 1 knows that whatever value x_2 may take, there is always some value x_1 he can choose that will induce player 2 to accept a share of $1 - x_1$ now rather than wait and get $1 - x_2$. Player 1 thus wishes to choose x_1 low enough for player 2 to accept it now rather than wait a period, incur the additional cost c_2, and have x_2 accepted. He can do this by choosing x_1 so that $(1 - x_1)$ is at least as big as $\delta_2[(1 - x_2) - c_2]$:

$$(1 - x_1) \geq \delta_2[(1 - x_2) - c_2] \tag{3}$$

Unless the resulting value of x_1 equals 1, giving him the entire pie, player 1 will want to choose x_1 to make inequality (3) an equation. This gives us player 1's reaction equation,

$$x_1(x_2) = Max\{1 - \delta_2[(1 - x_2) - c_2], 1\} \tag{4}$$

If $x_2 = 0$, player 1 will choose $x_1 = 1 - \delta_2 + \delta_2 c_2$, because he believes that if player 2's equilibrium offer is always to keep the entire pie, player 1 can still hold out for a small x_1 that player 2 will accept rather than waiting and making his own offer. As x_2 rises, $x_1(x_2)$ rises linearly at a rate of δ_2 as player 1 can get away with asking for a larger and larger share. When $x_2 = 1 - c_2$, player 1 responds with $x_1 = 1$, asking for the entire pie because player 2 will not benefit from waiting a period only to offer all but c_2 of the pie to player 1 anyway. Any higher values of x_2 also would cause player 1 to offer $x_1 = 1$ because player 2 would obtain even less benefit by waiting.

Now let us think about player 2's reaction curve. Taking player 1's next-period offer of x_1 as given, player 1 would like to offer $x_2 = 0$ if player 1 would accept it, but otherwise as little as possible. Player 1 will compare x_2 from accepting to $\delta_1(x_2 - c_1)$ from rejecting and making his own offer later. Equating these yields,

$$x_2 \leq \delta_1(x_2 - c_1) \tag{5}$$

Solving for x_1 yields player 2's reaction function:

$$x_2(x_1) = Min\{0, \delta_1(x_1 - c_1)\} \tag{6}$$

If $x_1 = 0$, player 2 can offer $x_2 = 0$ and player 1 will clearly do better to accept rather than wait and get the same zero share. If x_1 rises to $x_1 = c_1$, player 1 becomes indifferent about accepting or rejecting, and for higher x_1, player 2 must increase the value of his offer linearly at a rate of δ_1. When $x_1 = 1$, player 2 must offer $x_2 = \delta_1(1 - c_1)$ to get player 1 to accept the offer.

FIGURE 1:
REACTION CURVES x_1 AND x_2

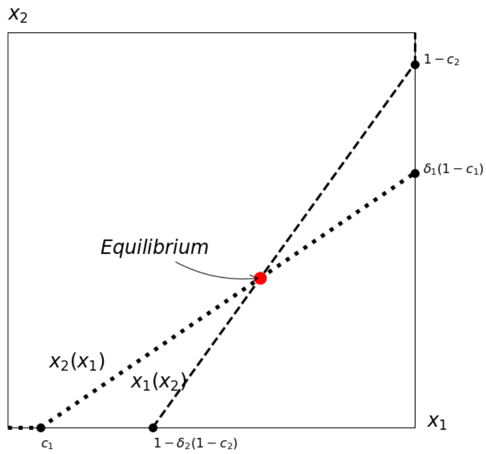


Figure 1 illustrates the reaction functions when $\delta_1 = \delta_2 = .7$ and $c_1 = c_2 = .08$. Note the intercepts on each axis that result in an interior solution, which requires:

$$c_1 < 1 - \delta_2(1 - c_2) \tag{7}$$

and

$$1 - c_2 > \delta_1(1 - c_1) \tag{8}$$

If these two reaction equations do have an interior crossing as in Figure 1, they solve to

$$x_1^* = \frac{1 - \delta_2 + c_2\delta_2 - c_1\delta_1\delta_2}{1 - \delta_1\delta_2} \tag{9}$$

and

$$x_2^* = \frac{\delta_1 - \delta_1\delta_2 + c_2\delta_1\delta_2 - c_1\delta_1}{1 - \delta_1\delta_2} \tag{10}$$

Rubinstein looks at the two cases of Model I, where $c_1 = c_2 = 0$ (but $\delta_1, \delta_2 < 1$, and they might be unequal) and Model II, where $\delta_1 = \delta_2 = 1$ (but $c_1 \geq 0, c_2 \geq 0$, and they might be unequal). In Model I, the reaction functions have a unique interior solution. In Model II, the reaction functions do not have an interior solution unless $c_1 = c_2$, in which case they have a continuum of solutions.

When both forces are at work in the present model, the reaction functions do intersect at a unique point, but they might not cross at an interior solution. This leaves open the possibility that with equal transaction costs, player 1's share is 100% because of a first-mover advantage. (It is easy to see that the first mover can earn at least some positive payoff because the second player does not want to incur the fixed offer cost). Thus, let us explore when the reaction curves do not cross, and instead intersect an an extremum.

FIGURE 2:
PLAYER 1 STRONGER: $x_1 = 1$

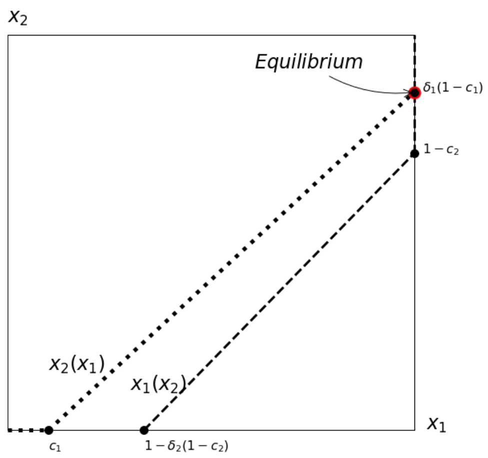
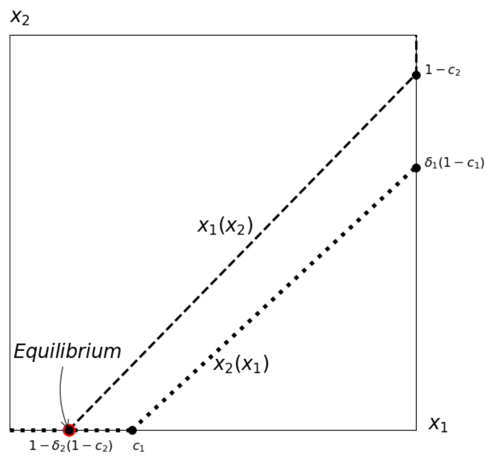


FIGURE 3:
PLAYER 2 STRONGER: $x_1 = 1 - \delta_2(1 - c_2)$



The reaction curves will not cross at an interior solution if $c_1 > 1 - \delta_2 + \delta_2 c_2$ (in which case player 2 gets almost the entire pie; $x_1 = 1 - \delta_2(1 - c_2)$) or if $\delta_1(1 - c_1) > 1 - c_2$ and $c_1 < 1 - 1/\delta_1 + \frac{c_2}{\delta_1}$ (in which case player 1 gets the entire pie; $x_1 = 1$). We can rule out $1 - \delta_2 + \delta_2 c_2 < 1 - 1/\delta_1 + \frac{c_2}{\delta_1}$, so it is not possible to have both corner solutions at once: multiple equilibria. (The previous sentence's inequality implies $\delta_2 + \delta_2 c_2 < 1/\delta_1 - c_2/\delta_1$, so $\delta_1(\delta_2 - \delta_2 c_2) < 1 - c_2$, which is false since δ_1 and δ_2 are less than 1.) Let us consider the salient cases.

(1) Suppose both players have the same positive fixed cost and the same positive discount factor, so If $\delta_1 = \delta_2 = \delta > 0$ and $c_1 = c_2 = c > 0$. In this case, illustrated in Figure 1, there is always a unique corner solution, not a continuum as happens in Model II, where just the offer costs are positive and equal. Here, $x_1^* = \frac{1-\delta_1+c_1\delta_1-c_1\delta_1^2}{1-\delta_1^2}$, which is less than 1. To understand this, first think of the intuition for why in Model II, with $\delta_1 = \delta_2 = 0$, it would be an equilibrium (one of many) for player 1 to offer $x_1 = 1$ and have it accepted. This is so because in that equilibrium it expected that player 1 will offer $x_1 = 1$ whenever it is his turn to offer, so the least player 2 can acceptably offer for player 1's share is $1 - c$, but in that case player 2, having incurred the cost c by refusing player 1's initial offer, has gained nothing by delay. If $\delta > 0$, on the other hand, then player 2 can offer something a little less generous than $x_1 = 1 - c$ and player 1 will still accept— so player 1 must forestall this by being a little more generous and set $x_1 < 1$.

(2) Suppose both players have the same fixed cost $c_1 = c_2 = c$, but different discount factors. In that case, there is an interior solution no matter how much their discount factors differ. A corner solution giving player 1 the entire pie would require, from (8), that $1 - c < \delta_1(1 - c)$, which is impossible. A corner solution giving player 2 almost the entire pie requires, from

(7), that $c > 1 - \delta_2 + \delta_2 c$, which is equivalent to $c > (1-\delta_2)(1) + \delta_2 c$, which is also impossible if $c < 1$.

(3) Suppose both players have the same discount rate, $\delta_1 = \delta_2 = \delta$, but different offer costs. This generates Figure 2's corner solution of $x_1 = 1$ if c_2 is big enough, and Figure 3's $x_1 = 1 - \delta_2(1 - c_2)$ if c_2 is small enough. Let's focus on the $x_1 = 1$ outcome, since it is possible for smaller differences in offer costs. Inequality (8) tells us that $x_1 = 1$ if $\delta > \frac{1-c_2}{1-c_1}$, i.e., if the offer costs and their difference are big enough relative to the amount of discounting. How big is big enough? An instructive special case is where the discounting cost per player is the same magnitude as the offer cost, so we can see which kind of cost as more impact on the qualitative outcome. Therefore, let's consider the case of $c_1 = .5(1-\delta)$, with $c_2 = \gamma c_1$ for some $\gamma > 1$. In that case, our inequality is, after simplification, $\gamma > \frac{2-\delta-\delta^2}{1-\delta}$. With even as extreme strong as $\delta = .5$, this says that player 2's offer cost would have to 2.5 times as big as player 1's to have an equilibrium in which player 1 asks for a 100% share.

Thus, the canonical Rubinstein's Model 1 is reasonably robust to the addition of fixed offer costs of bargaining, whereas Model 2 changes drastically with the addition of discounting. The key difference between discounting and fixed offer costs is that discounting has a higher absolute cost to a player who expects a greater bargaining share, whereas the fixed offer costs are independent of one's share. As a result, there is a strong tendency for discounting to drive the equilibrium towards an even split, because if one player's equilibrium share is larger, so is his cost of delay, which tends to reduce his equilibrium share. This tendency can be reversed only if the players' fixed offer costs are considerably different from each other.

References

Cramton, Peter C. (1991) "Dynamic Bargaining with Transaction Costs," *Management Science* , 37 (10): 1221-1233 (Oct. 1991).

Rubinstein, Ariel (1982) "Perfect Equilibrium in a Bargaining Model," *Econometrica*, 50 (1): 97-109 (Jan. 1982).

Calculations.

Solving,

$$1 = -\delta_2 c_2 + \delta_2 x_2^* + x_1^* \tag{11}$$

$$1 + \delta_2 c_2 - x_1^* = \delta_2 x_2^* \tag{12}$$

$$x_2^* = \frac{1 + \delta_2 c_2 - x_1^*}{\delta_2} \tag{13}$$

So

$$1 - \frac{1 + \delta_2 c_2 - x_1^*}{\delta_2} = \delta_1[-c_1 + x_1^*]. \tag{14}$$

$$\delta_2 - 1 - \delta_2 c_2 + x_1^* = \delta_2 \delta_1[-c_1 + x_1^*]. \tag{15}$$

$$x_1^* = -\delta_2 \delta_1 c_1 + \delta_2 \delta_1 x_1^* - \delta_2 + 1 + \delta_2 c_2 \tag{16}$$

$$x_1^*(1 - \delta_2 \delta_1) = -\delta_2 \delta_1 c_1 - \delta_2 + 1 + \delta_2 c_2 \tag{17}$$

$$x_1^* = \frac{-\delta_2 \delta_1 c_1 - \delta_2 + 1 + \delta_2 c_2}{(1 - \delta_2 \delta_1)} \tag{18}$$