Posted Prices vs. Haggling: The Economics of Isoperfect Price Discrimination

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Abstract

Standard discussions of perfect price discrimination rest on a hidden assumption: that the monopolist can make take-it-or-leave-it offers. If a monopoly charges different prices to each of a large number of buyers, the correct paradigm is not the ultimatum game, but bilateral monopoly. The monopolist's profit will not be the entire surplus, but something less. Under "isoperfect price discrimination"– a constant split $\lambda = .5$ of the bargaining surplus with each buyer— and constant marginal cost, the monopolist has the same profit as monopoly pricing if the demand curve is linear, less if demand is concave, and more if demand is convex. For constant-curvature demand functions, increased convexity increases the attractiveness of price discrimination. Upward sloping marginal cost tends to make the monopolist prefer price discrimination. Usually, but not always, isoperfect price discrimination is complemented by an idiosyncratic product design and informative advertising, whereas simple monopoly pricing is facilitated by plain-vanilla designs promoted via pure hype.

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Various May 3 notes.

1. If you want to have a showroom, hyou also want to use IPD.

2. Baye: what about incomplete info? If there is a chance of bargaining breakdown, then posted pricing is better with linear pricing.

3. If there is a chance of bargaining breakdown, the effect is like having a smaller λ for the seller (and, possibly, IPD is inefficient compared to posted pricing).

4. In the Supply section, talk about how sequential bargaining would lead to the buyers getting a share of that lower surplus triangle.

5. What about bargaining when the seller doesn't know the buyer's value? Will he just hcarge the posted monopoly price?

6. A sales force is expensive–worth mentioning as another tie-breaker. Maybe have an intro story: "Acme Carbuletors, the giant monopoly, has long had an army of salesmen who figure out the valuations of each potenital consumer and try to extract the maximum surplus from them. VP-Finance Smith says "They cost too much for the additional value they bring. Fire them all" VP-Marketing Jones says, "Their cost is small rlative to the value of the sales an the information teh bring to us. VP-Econmics Brown says, "In fact, even if they had zero salaries, we should fire them all. We'd do better with a single uniform prices for all cusotmers, regardless of their values." We will show the conditions udner which Brown is right".

It could be that the cost of hte sales force is a commission. That affects the net lambda, maybe. Or maybe not— if the commission is paid to increase lambda. This could use some thought. It wouldn't be technical modelling stuff, just tricky verbal discussion maybe.

7. Second-order PD can be quality OR quantity. We have the Mussa-Rosen section with upgrades. We could think about having consumers who each have their own individual demand curve, no buying just one unit. A good thing about IPD is that it allows price discrimination to extract more from each individual too. Maybe this will be in a second paper, with the upgrade stuff. Baye made this point.

8. Companies will have an incentive to have a reputation for Honesty, to have salesman disclose info that will increase dispersion, turning off as many customers as are encouraged to buy.

9. Remembe the ChangRiordan spokes world. Bar-Isaac AER too. Price and Variety in the Spokes Model, 1. Yongmin Chen1, 2. Michael H. Riordan2 The Economic Journal Volume 117, Issue 522, pages 897–921, July 2007 7.

10. I wrote a lot of margincomments in the text too. One thing I tried was using λ-perfect price discrimination for the general pd case and isoperfect for the 50-50 split. See what you think of the ring of it.

1. Introduction

We usually think of perfect price discrimination as yielding to the monopolist the entire gains from trade, the sum of what would be producer and consumer surplus in a competitive market. For this reason, a monopolist would always prefer perfect price discrimination to using a single price if information and resale possibilities permit it.

The idea is simple, and is useful in teaching students how the monopolist's ideal is not to reduce surplus, but to maximize it but then capture it entirely as profits. Consider what happens when a single monopoly monopolist with a constant marginal cost of c faces buyers $i = 1, ..., N$ with reservation prices $\overline{p_1}, ..., \overline{p_N}$, each buying one unit. In a competitive market, profit would be zero. Using standard monopoly pricing, the monopolist would choose a price which trades off a high profit margin against higher sales, and which would yield moderate profits while leaving surplus for the high-valuing buyers and destroying potential surplus because of the low-valuing buyers who drop out of the market. In our conventional model of perfect price discrimination, the monopolist charges prices $\overline{p_1}, ..., \overline{p_N}$, for a profit of $\sum_i^N (\overline{p_1} - c)$, and he captures the entire social surplus, which, however, is at its maximal level.

Everyone acknowledges that the standard model relies on strong assumptions that usually do not hold in the real world. The monopolist must be able to monitor the quantity sold or prevent resale, or the consumer with the lowest price will buy a large quantity and resell to all the other consumers. And the monopolist must be able to identify which consumer is which and know the reservation prices on the demand curve exactly, a very strong informational requirement. If these assumptions are valid, however, the perfect price discriminator will have higher profits than the monopolist who uses a single posted price.

What is not well recognized, though the point is simple enough, is that the standard model assumes that the monopolist can make take-it-or-leave-it offers, as in the "ultimatum game" so often used as the extreme case in bargaining theory.[1] If this is not true, the monopolist might be better off *not* being able to price discriminate. He may wish to commit to not knowing the reservation prices of individual consumers, or not being able to bargain with each of them separately. This is the main point of the present article: perfect price discrimination is not necessarily good for the seller. Sometimes it is, though, and most of our effort will be devoted to showing when.

We have moved most of our verbal discussion and literature review to later in the paper so that the reader may see the model first and thus pin down the idea more clearly. We will, however, discuss a little more here why we are dissatisfied with the standard model.

The standard model of perfect price discrimination is inconsistent with our usual story for what would happen if our one monopolist faced only one buyer, buyer 1. In that case, we would label the situation as bilateral monopoly. We would call the buyer a monopsonist, because if he were to disappear from the world, so would his demand for the product. Free entry of producers is a reasonable assumption, and thus we expect producers to have zero profits in equilibrium. Free entry of the owners of scarce resources such as labor, land, or minerals is not, nor is free entry of consumers. Owners of resources and owners of demand will earn long-run producer and consumer surplus in equilibrium, scarcity rents that competition does not eliminate. The consumer is a monopopsonist with respect to his own demand.

We are not sure as economists how to model bilateral monopoly. It is a bargaining situation, which, like oligopoly pricing, needs further information before we make predictions. In the absence of such information, most economists would predict that the consumer and producer would trade the efficient quantity and would split the surplus equally. In our example, the monopolist would then charge

[1] The modern mechanism design approach to price discrimination does recognize the need for take-it-or-leave-it offers, in the form of commitment to a mechanism, but it has bypassed perfect price discrimination, the simplest form.

the price $(\overline{p_1} - c)/2$ for the single unit buyer 1 desires. Although we would not be confident in this prediction for so unstructured a situation, but we certainly would not predict the price to be $\overline{p_1}$.

What is different in the situation modelled by perfect price discrimination? Only that instead of one bilateral monopoly, there are N of them, one for each buyer. The monopolist is a monopoly because of his uniqueness, but each buyer is a monopsonist because he is the unique source of his own demand. The monopolist's profit would not be $\sum_i^N(\overline{p_1} - c)$, but only half of that amount, if each side is equally good at bargaining, or a fraction λ of that amount if the monopolist is better or worse. This is what we will call "λ-perfect price discrimination" or "isoperfect price discrimination" ("iso-" for "equal") if $\lambda = .5$.

2. The Model

A monopolist's marginal cost at output z is $c(z)$. Demand arises from a unit mass of consumers, where a consumer's willingness to pay θ for a single unit of the product is drawn from a distribution $F(\theta)$ with positive density $f(\theta)$ over its support. Thus, the price p yields demand quantity $z(p) = 1 - F(p)$. We will refer to the inverse function $p(z)$ as "the inverse demand function" or "the demand curve", which by our positive density assumption falls monotonically in z.

The monopolist may sell either by posting a single price or by bargaining with individual consumers. The result of bargaining is to split the surplus, the difference between the marginal cost and the reservation value, with fraction λ going to the monopolist and $(1 - \lambda)$ to the consumer.

All functions and parameters are common knowledge. The monopolist knows each consumer's reservation price, can identify each consumer, and can prevent resale.

Let us define terms as follows:

Definition: Under **"monopoly pricing"** : the monopolist posts a single price, which buyers may only accept or reject.

Definition: Under **"Pigouvian perfect price discrimination"** : the monopolist bargains with each buyer separately, and captures the entire surplus.

Definition: Under **"λ-perfect price discrimination"** : the monopolist bargains with each buyer separately, and captures fraction λ of the surplus from each buyer.

Definition: Under **"isoperfect price discrimination" or "IPD"** : the monopolist bargains with each buyer separately, and captures half the surplus from each buyer. i.e., $\lambda = .5$.

We use the term "λ-perfect price discrimination" because our version of perfect price discrimination does not give the bargaining power to either buyer or seller, but treats them as both benefiting from a bargain which perfectly achieves the gains from trade. Their shares are not strictly equal unless bargaining powers are equal, but they both get positive shares so long as λ does not equal zero or one. The special case of $\lambda = .5$ is important enough that we have assigned it its own term, "isoperfect price discrimination".

We have just described a reduced-form model of the pricing process. This flows easily from natural structural models. The monopolist engages in simultaneous discrete-time alternating-offers bargaining games with all the consumers, in the style of Rubinstein (1982).[2] In odd periods the monopolist makes simultaneous offers and each consumer accepts or rejects the offer made to him if he has not yet accepted any offer, having observed offers made to other consumers but not whether they have accepted or not. All players then observe which offers were accepted and in the even periods the consumers make simultaneous offers to the monopolist, who decides which offers to accept. Let the discount rates be ρ_M and ρ_C for the monopolist and the consumers. The equilibrium outcome is for all offers to be accepted in the first period and for the offer to split the surplus as follows.

Allowing the time interval between the alternating offers to shrink, the monopolist extracts a fraction $\lambda \in (0, 1)$ of the surplus from the relationship, where λ is determined, in the usual way, by the relative patience of the bargaining parties. If he sells z^* units, the haggling monopolist earns a profit from consumer θ of

$$\lambda[p(z) - c(z^*)] \quad \text{where} \quad \lambda \equiv \frac{\rho_C}{\rho_C + \rho_M}. \tag{1}$$

In choosing between the two feasible sales strategies the monopolist faces a simple trade-off. Haggling with consumers has the disadvantage of ceding some

[2] We describe a bargaining game here in which the monopolist bargains with all consumers simultaneously. We could instead have used a model in which alternating-offer games were played with consumers in an exogenous sequence or in a sequence chosen by the monopolist.

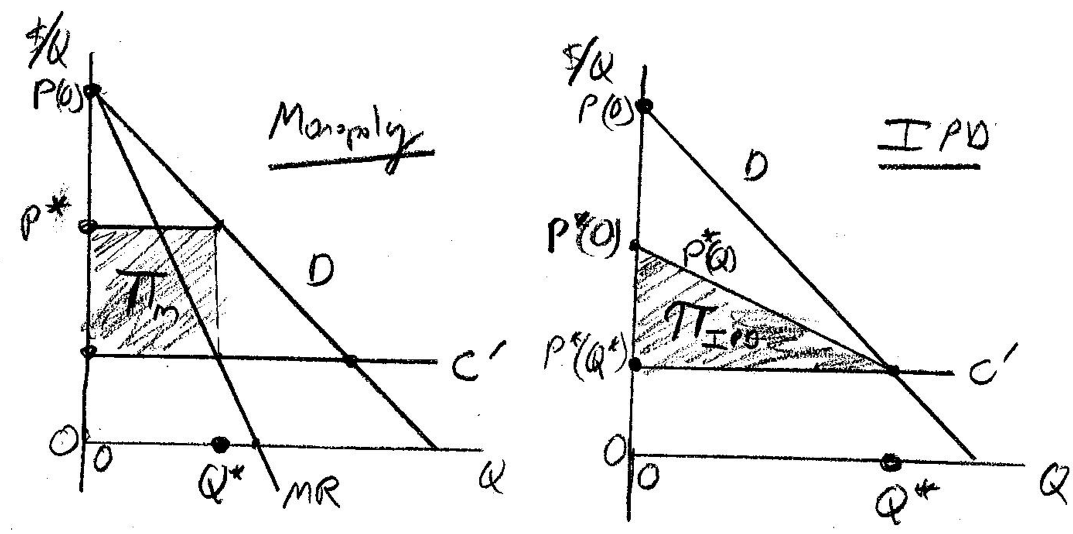


FIGURE 1. Linear Demand and Constant Marginal Cost

of the surplus to consumers, whereas a posted-price system fully commits him and gives him all the bargaining power. However, haggling allows the monopolist to tailor the transaction price to the willingness-to-pay of each consumer; that is, he is able to engage in first-degree price discrimination. The haggling strategy is attractive, therefore, whenever the monopolist stands to gain from such discrimination. This is so when the profit earned from the familiar uniform-price sales strategy is a only small fraction of the maximum feasible surplus; this fraction is precisely the right-hand side of equation (1). Of course, this fraction depends upon the characteristics of consumer demand (via the distribution $F(\cdot)$) and the production technology (via the marginal cost c); it represents, essentially, the monopoly power of the supplier. Hence, a determination of when a haggling sales strategy is optimal boils down to this question: when is the power of a uniform-pricing monopolist relatively weak?

If we do not say otherwise, our propositions will apply to a "basic model" in which transaction costs are zero, bargaining power is equal, marginal cost is constant, demand is linear, and consumers are fully informed of their tastes: $\lambda = .5$, $c(z) = c$, and $p''(z) = 0$. We will relax each of these assumptions later.

Proposition 1: *In the basic model with linear demand and constant marginal cost, the monopolist is indifferent between monopoly pricing and isoperfect price discrimination, earning profits in each case equal to half of the total surplus.*

Proof. Let demand be linear: $p(z) = a - bz$. Profit from quantity z using monopoly pricing is $z(p-c) = az - bz^2 - cz$, which is maximized when $a - 2bz - c = 0$ so $z = \frac{a-c}{2b}$ so $p = a - \frac{a-c}{2} = \frac{a+C}{2}$ and profit is $z(p-c) = \frac{a-c}{2b}(\frac{a+c}{2} - c) = \frac{a-c}{2b}\frac{a-c}{2} = \frac{(a-c)^2}{4b}$.

Under isoperfect price discrimination, quantity is chosen to maximize surplus, so the last reservation value served equals $a - bz = C$ and $z = \frac{a-c}{b}$. Total surplus is then $(1/2)(a - c\frac{a-C}{b} = \frac{(a-c)^2}{2b}$.

If the monopolist's bargaining power is $\lambda = .5$, then his profit is half of the surplus, or $\frac{(a-c)^2}{4b}$, the same as for monopoly pricing. $\square$

Proposition 1 combines the folk theorem that a monopolist facing linear demand can capture half the social surplus with the idea that under isoperfect price discrimination the same will be true. Proposition 1 establishes that even with perfect information and no resale it is not necessarily to a monopolist's advantage to have a different price for each buyer. In addition to those advantages, he needs high bargaining power. In Pigouvian perfect price discrimination, he has complete bargaining power– $\lambda = 1$ in our notation. If bargaining power is symmetric, then if demand is linear the monopolist loses just as much from the ability of high-value consumers to bargain down the price as he gains from being able to sell to low-valuing consumers who would be shut out if a posted price were used.

Proposition 1 covers the base case, where the assumptions balance so that perfect price discrimination and monopoly pricing yield identical profits. In the rest of the paper we will look at what happens when we change the assumptions in various ways.

Lemma 1 notes the existence of an indifference level λ^* of bargaining power, even under general conditions of demand and supply.

Lemma 1: Let the marginal cost and demand curves take any desired shape. There exists some bargaining strength λ^* for the monopolist just big enough that he is indifferent between posted monopoly pricing and λ-perfect price discrimination.

Proof. Set $\lambda = 0$. The monopolist will then prefer monopoly pricing, since IPD yields him zero profits. Now set $\lambda = 1$. A monopolist using IPD will set the price above the monopoly price p_m for all consumers on the portion of the demand curve above p_m, earning more profits from them than would a simple monopolist, as well as earning profits from prices above marginal cost for consumers further down on

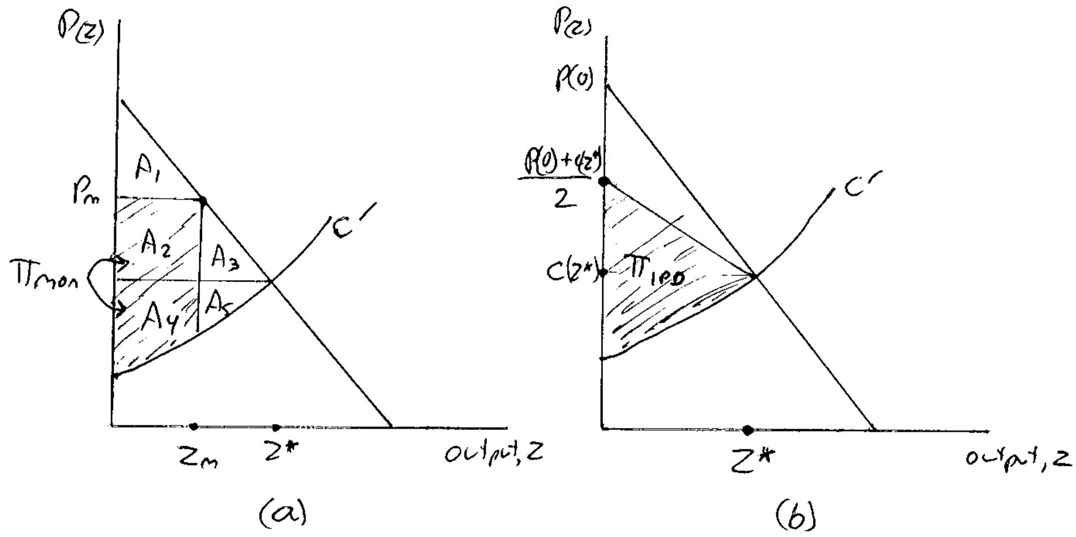


FIGURE 2. Linear Demand and Rising Marginal Cost

the demand curve. Profits from IPD rise in λ, so there must be some interior level λ^* that makes IPD and monopoly pricing equally attractive. $\square$

3. The Shape of the Marginal Cost Curve

Let us now retain the assumption of linear demand but allow marginal cost to rise or fall. Rising marginal cost, the standard case, makes isoperfect price discrimination superior to posted monopoly pricing.

Proposition 2: *If demand is linear and marginal cost is increasing ($c'(z) > 0$), the monopolist prefers balanced isoperfect price discrimination to monopoly pricing. If marginal cost is decreasing ($c'(z) < 0$), he prefers monopoly pricing.*

Proof. Under isoperfect price discrimination, the equilibrium marginal cost would equal the value of the lowest customer served, so output would be z^* and marginal cost $c(z^*)$ such that $P(z^*) = c(z^*)$ as in Figure 2. Prices would range from $c(z^*)$ for the marginal consumer up to $\frac{P(0)+c(z^*)}{2}$ for the highest-valuing consumer. Profit, from Proposition 1, would be half of triangle $A_1 + A_2 + A_3$, plus, since marginal cost is rising, areas A_4 plus A_5.

Under posted monopoly pricing, output would be set so that marginal cost equalled marginal revenue, so output would be some amount z_m which is less than IPD's z^*. This output would be greater than the monopoly output of z_1 that would maximize profit if the marginal cost curve were flat at $c(z^*)$. From Proposition 1 we know that that when output is z_1 and marginal cost is constant, profit is half of triangle $A_1 + A_2 + A_3$, and that profit from any other output would be less. Thus, rectangle A_2 is less than half the area of triangle $A_1 + A_2 + A_3$.

The profit from monopoly pricing is $A_2 + A_4$. The profit from IPD was $.5(A_1 + A_2 + A_3) + A_4 + A_5$. We have just established that $A_2 < .5(A_1 + A_2 + A_3)$, so since $A_4 < A_4 + A_5$, profits are higher from IPD for rising marginal cost.

Now, for declining marginal cost, refer to Figure 3. The profit from IPD is half of the area of the triangle lying between the $c = c(z^*)$ line and the demand curve minus areas $(B_1 + B_3 + B_6 + B_7)$, which are parts of cost as well as of revenue. By Proposition 1, half of the area of the triangle lying between the $c = c(z^*)$ line and the demand curve equals the profit from monopoly if marginal cost were that low, which is $(B_3 + B_4 + B_5 + B_6)$. Thus,

$$\begin{aligned} \pi_{IPD} &= (B_3 + B_4 + B_5 + B_6) - (B_1 + B_3 + B_6 + B_7) \\ &= B_4 + B_5 - B_1 - B_7 \end{aligned} \tag{2}$$

The profit from monopoly given the actual declining marginal cost curve is

$$\pi_m = B_2 + B_4. \tag{3}$$

Thus, we need to prove that

$$\pi_m - \pi_{IPD} = (B_2 + B_4) - (B_4 + B_5 - B_1 - B_7) = B_2 - B_5 + B_1 + B_7 > 0. \tag{4}$$

A useful fact is that given the actual declining marginal cost curve, output of z_m instead of z_2 maximizes profit, so

$$\begin{aligned} \pi_{m,Q_m} &> \pi_{m,Q_2} \\ B_2 + B_4 &> B_4 + B_5 - B_1 \\ B_2 - B_5 + B_1 &> 0 \end{aligned} \tag{5}$$

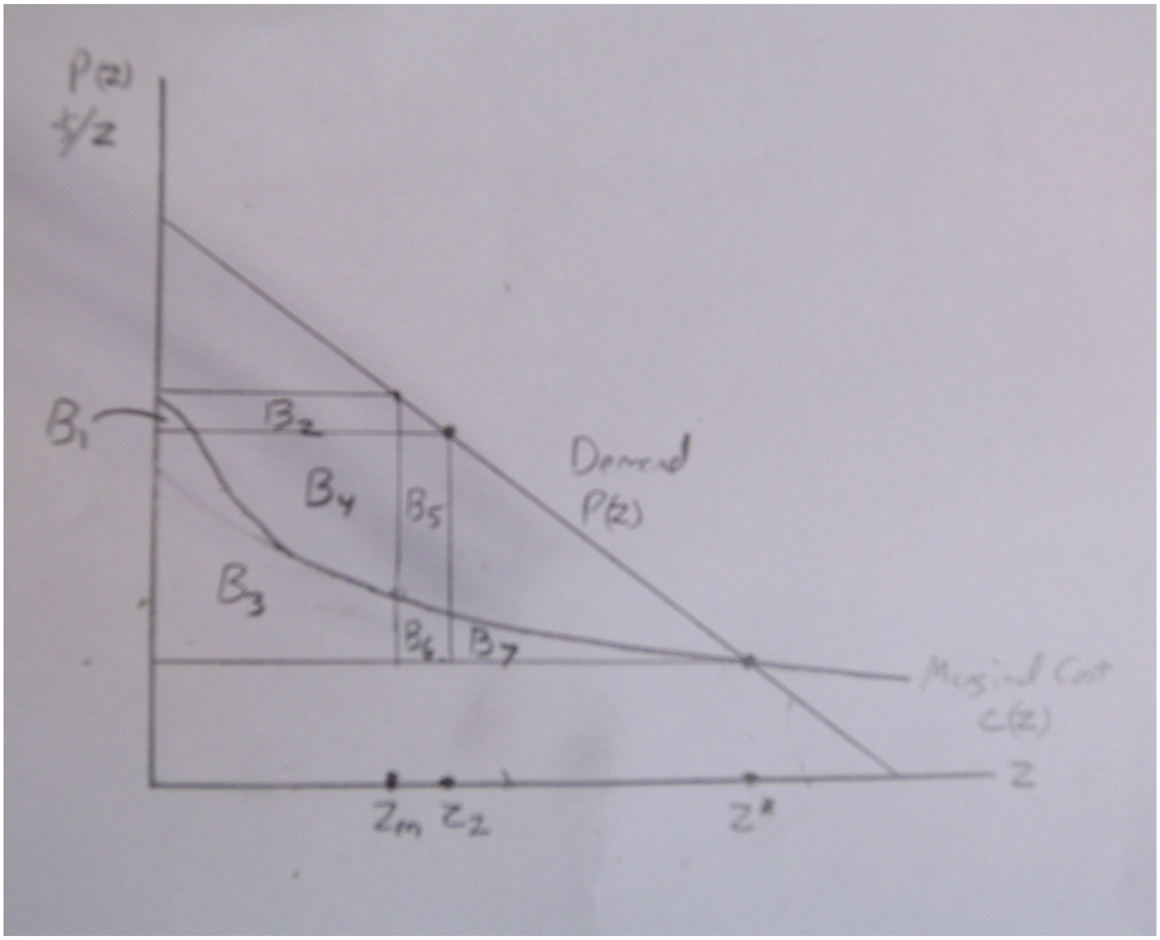


FIGURE 3. Linear Demand and Declining Marginal Cost

Inequality (5) implies that Inequality (4) is true, since $B_7 > 0$. Thus, monopoly profits are indeed higher than IPD profits when marginal cost is falling.[3] □

Proposition 2 shows that bargaining has a special advantage for the monopolist when there are multiple buyers. Each buyer is marginal, so the monopolist's marginal cost is high with each of them, and the surplus to be bargained over with each does not include his inframarginal profits. If bargaining broke down with a positive measure of buyers then the monopolist's marginal cost would be lower, but that possibility is irrelevant in a Nash equilibrium, since it would require deviations from equilibrium behavior by multiple players. Thus, under bargaining

[3]xxx This proof is OK for the curves drawn, but the curves could be drawn so other areas exist, or some of these areas do not, so it is not rigorous yet. We might want an algebraic proof instead, as being possibly simpler.

price discrimination the monopolist can retain the entire amount of what would be producer surplus under marginal-cost pricing, since it is all inframarginal surplus.

There is a paradox when a monopolist with increasing marginal cost uses bargaining with many consumers: it seems that he has incentive to drive output beyond the efficient level in order to increase his marginal cost and thereby improve his bargaining position. Suppose the monopolist's marginal cost is 10 at the efficient output level of 200, but that if he sold 201, his marginal cost would rise to 12. Suppose too that at a price of 12 he would sell 199 units. At the efficient output level, part of the surplus he split with buyers was the difference between 10 and 12 times 199, and buyers received 199 of that if the split was 50-50. Now, at the cost of having to sell the 200th and 201st unit at a loss, the monopolist captures that 199.

The way to resolve the paradox is to think about what kind of commitment would achieve it. If he precommits to produce 201 units, then his marginal cost is 0 at the time of bargaining. He can't get out of his commitment, so his opportunity cost of the sale is 0 (if there are no alternative buyers) or the reservation price of a consumer further down on the demand curve, which will be lower than the efficient output's marginal cost. Therefore, precommitment does not raise the monopolist's marginal cost; it reduces it, hurting his bargaining position.

Even if $\lambda = 0$, so the monopolist has zero bargaining power, he will still have positive profits under bargaining if marginal cost is upward sloping. The price would equal marginal cost for all consumers, since each consumer captures the entire bargaining surplus. Since marginal cost exceeds average cost, however (assuming, as we do, that there is no fixed cost), the monopolist will still have revenues greater than his costs. There is a crucial difference between the monopolist negotiating with each buyer separately— as we assume here— and the monopolist negotiating with the buyers as a group. Negotiating with them as a group, the disagreement point would be zero profits for the monopolist, and so if he has zero bargaining power he would retain zero profits. Negotiating with them as individuals, the disagreement point for each negotiation is that the other negotiations all succeed, so the marginal cost is equal to the level where the marginal cost curve crosses the demand curve. Bargaining yields positive profits even with zero bargaining power because the monopolist profits from the inframarginal sales. These positive profits, however, will be less than the monopoly profit, since setting the price equal to marginal cost yields lower profit than setting it equal to the monopoly price (or equal, if the monopoly price equals marginal cost because marginal cost is vertical at a low level).

4. History and Literature

Having laid out a model, we can now better discuss why we think it is important. Economists often assume that a large monopolist is able to make take-it-or-leave-it offers, but this ability, if it exists, is independent of size. A buyer is not a price taker simply because he is small relative to the monopolist. That is to confuse the ability of a buyer to influence the competitive market price or the single monopoly price in a large market with his bargaining position in one-on-one bargaining. The buyer may indeed be too small to much affect the market, but in one-on-one bargaining he represents one hundred percent of demand. If the monopolist finds it to his interest to reduce his price to sell to a particular buyer, he will do so. When the monopolist chooses one price for each buyer, buyer and monopolist are symmetrically placed. If it worthwhile for the monopolist to make a special offer to this buyer, so too is it worthwhile for that buyer to make a special counteroffer to the monopolist. Perfect price discrimination reduces them to equals in a tiny submarket.

The equivalence, ceteris paribus, of perfect price discrimination and bilateral monopoly goes unrecognized even by well-known economists. Jean Tirole has written in his *The Theory of Industrial Organization* (p. 135):

> "*First-degree* price discrimination is perfect price discrimination–the producer succeeds in capturing the entire consumer surplus. This occurs, for instance, when consumers have unit demands and the producer knows exactly each consumer's reservation price and (if these reservation prices differ) can prevent arbitrage between consumers. It then suffices for the producer to charge an individualized price equal to the consumer's reservation price."

David Kreps has written in his *A Course in Microeconomic Theory* (p. 306)

> The absolutely best position for a monopoly to be in is (a) to know the precise utility function of every consumer, (b) to be able to tailor a "price schedule" for each individual consumer, and (c) to be able to control absolutely any resale of the good being sold. Then the monopoly can make a

"take-it-or-leave-it" offer to each individual consumer, which extracts from the consumer all the surplus that this consumer would otherwise obtain from consumption of the good in question."

Even one of us has said, in *Games and Information* (p. 296):

> "**3 Perfect Price Discrimination.** This combines interbuyer and interquantity price discrimination. When the monopolist does have perfect information and can charge each buyer that buyer's reservation price for each unit bought, Smith might end up paying $50 for his first hot dog and $20 for his second, while next to him Jones pays $4 for his first and $3 for his second."

Arthur Pigou's 1920 book *The Economics of Welfare* , set out our modern paradigm for perfect price discrimination. He lays out the three "degrees" of price discrimination of which perfect price discrimination is the first. Pigou recognizes the problem of one-on-one bargaining, and even sees the connection to bilateral monopoly. In chapter XVI, pages 247-248 (in the 1932 2nd edition) he says:

> When a degree of non-transferability, of commodity units on the one hand, and of demand units on the other hand, sufficient to make discrimination profitable, is present, the relation between the monopolistic monopolist and each buyer is, strictly, one of bilateral monopoly. The terms of the contract that will emerge between them is, therefore, theoretically indeterminate and subject to the play of that "bargaining" whose social effects were analysed at the end of Chapter VIII. When a railway company is arranging terms with a few large shippers, the indeterminate element may have considerable importance. **Usually, however, where discrimination is of practical interest, the opposed parties are, not a single large monopolist and a few large buyers, but a single large monopolist and a great number of relatively small buyers. The loss of an individual customer's purchase means so much less to the monopolistic monopolist than to any one of the many monopolistic purchasers that, apart from combination among purchasers, all of them will almost certainly accept the monopolistic monopolist's price. They will recognize that it is useless to stand out in the hope of bluffing a concession, and will buy what is offered, so long as the terms demanded from them leave to them** ***any*** **consumers' surplus.** [our boldfacing] In what follows I assume that the customers act in this way. So assuming, we may distinguish three degrees of discriminating power, which a monopolist may conceivably wield. A first degree would involve the charge of a different price against all the different units

> of commodity, in such wise that the price exacted for each was equal to the demand price for it, and no consumers' surplus was left to the buyers. A second degree...

Pigou is aware of the problem, but he does not deal with it adequately. His argument for why a small monopolist will meekly accept the offer of the large monopolist relies on our intuition that the price is set for a large market of which that monopolist is only a small part. In the context of price discrimination, however, that monopolist is not a small part of the market to which that price is being charged, but the entire market.[4]

What is the source of monopoly power? Economists have evolved considerably in their thinking on this issue. The naive answer is that if a monopolist is large and buyers are small, the monopolist can set his own price rather than take a market price as given, and this large size is the source of monopoly power. That is not quite right: it is not size alone, but size relative to the industry that matters, and since a large firm in a large industry may have very little control over prices. In this view, it is concentration that matters, not size. Going a little further, one might add a caveat regarding entry: even if a firm is alone in its market, if entry is easy, then it may have no market power; barriers to entry are the key to monopoly power. This is the lesson of the contestability literature. Or, even if the monopolist is alone in a market and entry is impossible, it may be that close substitutes to its product exists, so it can raise the price by very little; lack of potential substitutes for one's product is the source of market power. This was the argument that saved Dupont from anti-trust sanctions in the famous *Cellophane* case.[5] All of these may be combined in the answer that the source of market power is the inelasticity of the demand curve facing the monopolist.

In this article, we would like to go an additional step: although inelasticity in the demand curve is the first qualification for monopoly power, it is not so powerful a generator of profits as is usually supposed. Just as important is bargaining power. We standardly assume, without much thought, that the monopolist can

[4]Pigou's argument works better if there exist many individuals of each consumer type, e.g., 10,000 consumers with reservation price $10.21, 8,900 consumers with reservation price $10.22, and so forth. Then one could construct a model in which because of transaction costs the monopolist finds it profitable to charge many different prices but no one consumer finds it worthwhile to haggle. Even there, however, the issue is not size, but size relative to the degree of discrimination.

[5]United States v. E. I. du Pont de Nemours, 351 U.S. 377 (1956).

make a take-it- or-leave-it offer to the buyers. If he cannot, our standard theory of monopoly is incorrect.

Economists have not looked at this is a problem of price discrimination, but some have studied how bargaining differs from posted pricing. Wang (1995) compares posted pricing to a particular bargaining process. The monopolist sets a reserve price. Then the buyer arrives, and the bargaining price is half the difference between the monopolist's reserve price and the buyer's value. We use perfect information, and no reserve price.

There is a large literature on posted pricing versus bargaining. See Wang (1995, EER). Gill & Thanassoulis, Anderson & Renault (2003) JET, Camera & Delacroix (2004) *Review of Economic Dynamic.*, Cason, Friedman, and Milam, (2003) IJIO. Desai & Purohit (2004) *Marketing Science.* In particular, note that Adachi (1999) addresses the problem in the context of a Hotelling duopoly model and finds that posted pricing tends to drive out negotiated pricing.

Riley & Zeckhauser (1983) started the literature on bargaining versus negotiation. Bulow & Klemperer (1996) looked at the effect of adding sellers; here we consider only the monopoly case.

5. The Shape of the Demand Curve

We have seen that linearity of the demand curve exactly balances the advantages of IPD and monopoly pricing. In this section we will see the effects of curvature and vertical drops in the demand curve. We will start with convexity and concavity generally, and then look more closely at demand with a constant degree of curvature to see how changing that degree affects the advantage or disadvantage of IPD.

Concavity, Linearity, and Convexity

Let us begin with concave vs. convex demand. We will define it as follows so as to exclude linear demand curves from being concave or convex but so as to include demand curves that are piecewise linear or have vertical drops at some price above marginal cost.

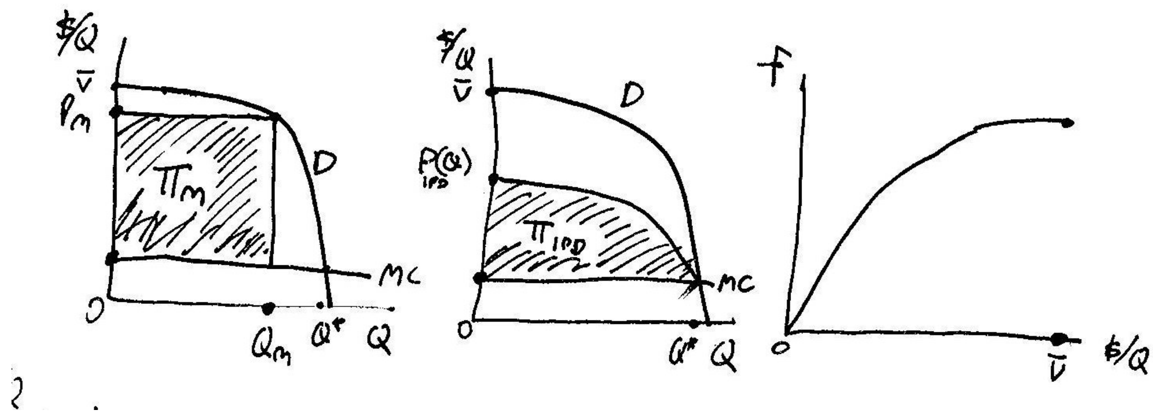


FIGURE 4. Profits from Concave Demand Are Highest with Posted Monopoly Pricing, Not IPD, and the Density of Demand is Higher for Higher Reservation Prices

Definition: Demand is **concave** *if no line segment connecting two points on the demand curve (excluding points above $\overline{v}$) contains any points lying above the demand curve, and at least one line segment connecting two points on it contains points lying below the demand curve.*

Maybe we don't need the concavity and convexity definitions. We are using the standard definitions if we say it is weak concavity and convexity and that linear is both. But maybe we should keep these for clarity on weak vs. strong and linear.

Definition: Demand is **convex** *if no line segment connecting two points on the demand curve contains any points lying below the demand curve, and at least one line segment connecting two points on it contains points lying above the demand curve.*

If the demand curve is differentiable, then $p' \leq 0$, and the curve is concave if $p'' < 0$, convex if $p'' > 0$. These definition of concavity and convexity say that a linear curve is neither concave nor convex. A linear spline might be; for example, a demand curve that is linear down to a particular reservation value and then vertical is concave under our definition.

Figure 4 shows concave demand, and gives the flavor of why monopoly pricing yields higher profits than isoperfect price discrimination for that shape of an inverse demand curve.

Put MC in that diagram. We should take out the density function for values.

Remark *If marginal cost is constant and the inverse demand curve is concave for prices greater than marginal cost, profits are greater from posted monopoly pricing than from isoperfect price discrimination. For convex demand, IPD is preferred.*

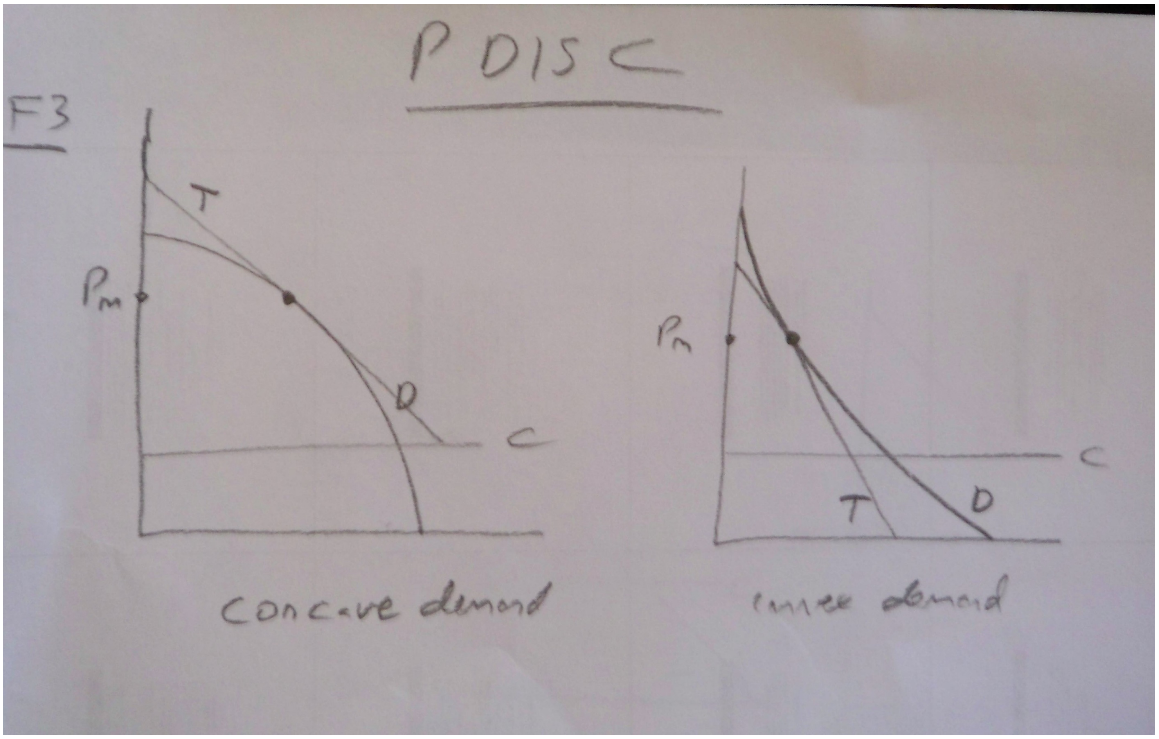


FIGURE 5. Convex and Concave Demand

This remark will be a direct corollary of a later, more general proposition, but it may be educational to see how this simpler statement can be proved geometrically. Pick the pair $(z, p(z))$ that maximizes profit. Draw a tangent T to the inverse demand curve at that point, as in Figure 5. We know from our earlier proposition that the posted pricing monopoly profit is half of the area of the first-best surplus from an inverse demand curve equal to T.

Since the inverse demand curve is concave for prices greater than marginal cost, the tangent T lies above the inverse demand curve, and strictly above for some prices. Thus, half of the first-best surplus from the inverse demand curve $p(z)$ is less than half of the first-best surplus if demand were the tangent T and so is less than half of the price from the posted price p_m. Hence bargaining is less profitable than posted pricing.

Similarly, if the inverse demand curve is convex for prices greater than marginal cost, the tangent T lies below the inverse demand curve, and strictly below for some prices. Thus, half of the first-best surplus from the inverse demand curve $p(z)$ is more than half of the first-best surplus if demand were the tangent T and so is more than half of the price from the posted price P_m. Hence bargaining is more profitable than posted pricing.

If demand is convex, many consumers have low reservation prices relative to the number of consumers with high prices. Concave demand indicates that high reservation prices are the most common. For a single-price monopolist, having lots of similar high-reservation price consumers is more important than having a lot of similar low- reservation price consumers. For a price discriminator, having more high-reservation-price consumers is desirable, of course, but not quite so important. He can capture the surplus of even a few high-reservation- price consumers, whereas the simple monopolist cannot.

Another way of putting this is that under asymmetric information and a take-it-or-leave-it offer, informational rents to each high-value consumer are larger if there are fewer of them.

This point holds for perfect price discrimination too. The perfect price discriminator always makes more than the simple monpolist, but he makes a lot more if the demand curve is convex.

The Degree of Convexity of the Demand Curve

To further discuss the influence of the shape of the demand curve, we need a way to describe the shape more finely. We wish to go beyond saying a curve is convex, concave, or neither. Therefore, let us think of a reasonable definition of "more convex". This will be a binary relation that implements a partial ordering, since we will not try to say that one curve is, for example, 2.5 times as convex as another, and we will not insist that any two curves can be ranked by convexity.

Consider the curves in Figure 6. How should we rank them in terms of convexity? It seems reasonable to say that the convex demand curve is more convex than the two linear demand curves, that those two curves are equally convex, and that those two are more convex than the concave demand curve. But how should we rank the two convex demand curves?[6]

The standard definition of "convex" for a differentiable weakly decreasing curve $p(x)$ is that $p''(x) > 0$. It would not make sense to use the size of p'' to

[6]Redraw this diagram to have one curve neither concave nor convex nad one curve nondifferentiable.

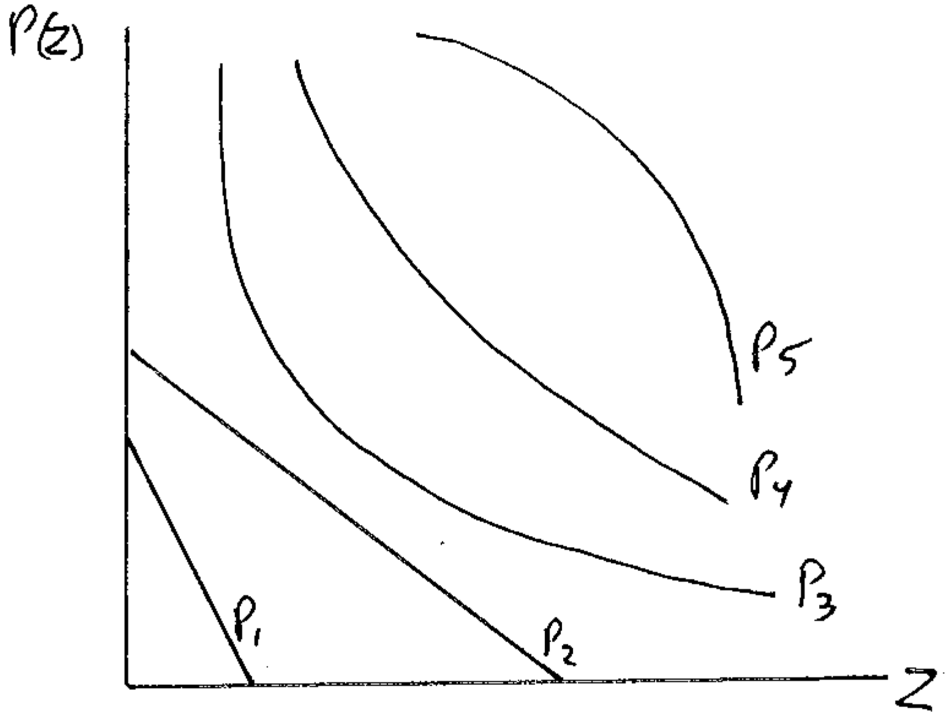


FIGURE 6. Which Curves Are More Convex?

define "more convex", however, because p'' is not unit-free. For example, if we counted prices in pennies instead of pounds, the demand curve $p = 1 - log(x)$ would change to $p = 100 - 100log(x)$, and its second derivative would change from $1/x^2$ to $100/x^2$, yet we do not want to say that the penny demand curve is more convex. We want a sort of topological definition instead, one that preserves the partial ordering through linear transformations and is thus unit free and scale free.

Definition. The differentiable demand curve $p(x)$ is **more convex** *than $r(x)$ if*

(1) *There is an x such that $p(x) = r(x)$ and $p'(x) = r'(x)$ then $p''(x) \geq r''(x)$.*
(2) *For any $\alpha > 0$ and $\beta > 0$, the function $\hat{p}(x) = \alpha p(\beta x)$ satisfies condition (1) with respect to $r(x)$.*

We want to cover non-differentiable functions too. Monotonic functions are continuous and differentiable almost everywhere, so there are right- and left- derivatives almost everywhere. Michael Rauh suggetsed looking at Arrow and Pratt, and saying that P1 is less convex than P2 if (1) We can move P1 to

Condition (1) says that if the two functions coincide in location and slope then the more convex function must have a bigger second derivative. Condition (2) says that the units of measurement do not matter; if the two functions do not coincide in location and slope but we change the units so that they do, the more convex function must have a bigger second derivative.

Note that we have not assumed that either p or r is convex in the first place. That still depends on the sign of p'' and r'' in the usual way. Possibly $p'' < 0$ and $r'' < 0$, but p'' is less negative than r''. Both functions are concave in that case, but p'' is more convex, or, extending the definition in the natural way, "less concave". It will be true, however, that to be ordered by our partial ordering, a demand curve must be concave, convex, or linear.

Our objective here is to make two demand curves comparable in scale, so that a small market's demand curve can be compared to a large market's, or one measured in dollars and ounces can be compared to one measured in euros and kilograms. One way to think of this is as transforming p1 so that (a) the transformed p1 still has the same ratio of posted price profit to social surplus, and (b) the transformed p1 has the same posted price profit as p2 does. Then we can answer whether the original p1 has bigger monopoly profit relative to social surplus than p2 does. Note that we can disregard marginal cost in this comparison— the social surplus is the excess of consumer values over our constant marginal cost, so we are in effect just looking at the portions of the demand curve above marginal cost.

Lemma 2. *Consider any two downward-sloping differentiable curves $p(x)$ and $r(x)$. Suppose that there is an $x_L < x_H$ such that $p(x_L) = r(x_L)$ and $p(x_H) = r(x_H)$ so that the two curves intersect at those two points, and that $p(x) > r(x)$ for $x_L < x < x_H$. Then it cannot be the case that $p(x)$ is more convex than $r(x)$.*

Proof. See Figure 7, which depicts two curves, $p(x)$ and $r(x)$ which intersect twice with $p(x) > r(x)$ between the intersection points. The curve $r(x)$ must have a steeper slope than $p(x)$ at x_L and a gentler slope at x_h since $p(x) > r(x)$ in between.[7] As a result, there must exist at least one ray through the origin R on which the slopes are equal: $p'(R_p) = r'(R_r)$.

Suppose p is more convex than r. Our definition says that this implies that for any transformation $\hat{p}(x) = \alpha p(\beta x)$, if we can find some x such that $\hat{p}(x) = r(x)$ and $\hat{p}'(x) = r'(x)$ then $\hat{p}''(x) \geq r''(x)$. The transformation $\hat{p}(x) = \alpha p(\beta x)$ shrinks every

[7] xxx make rigorous

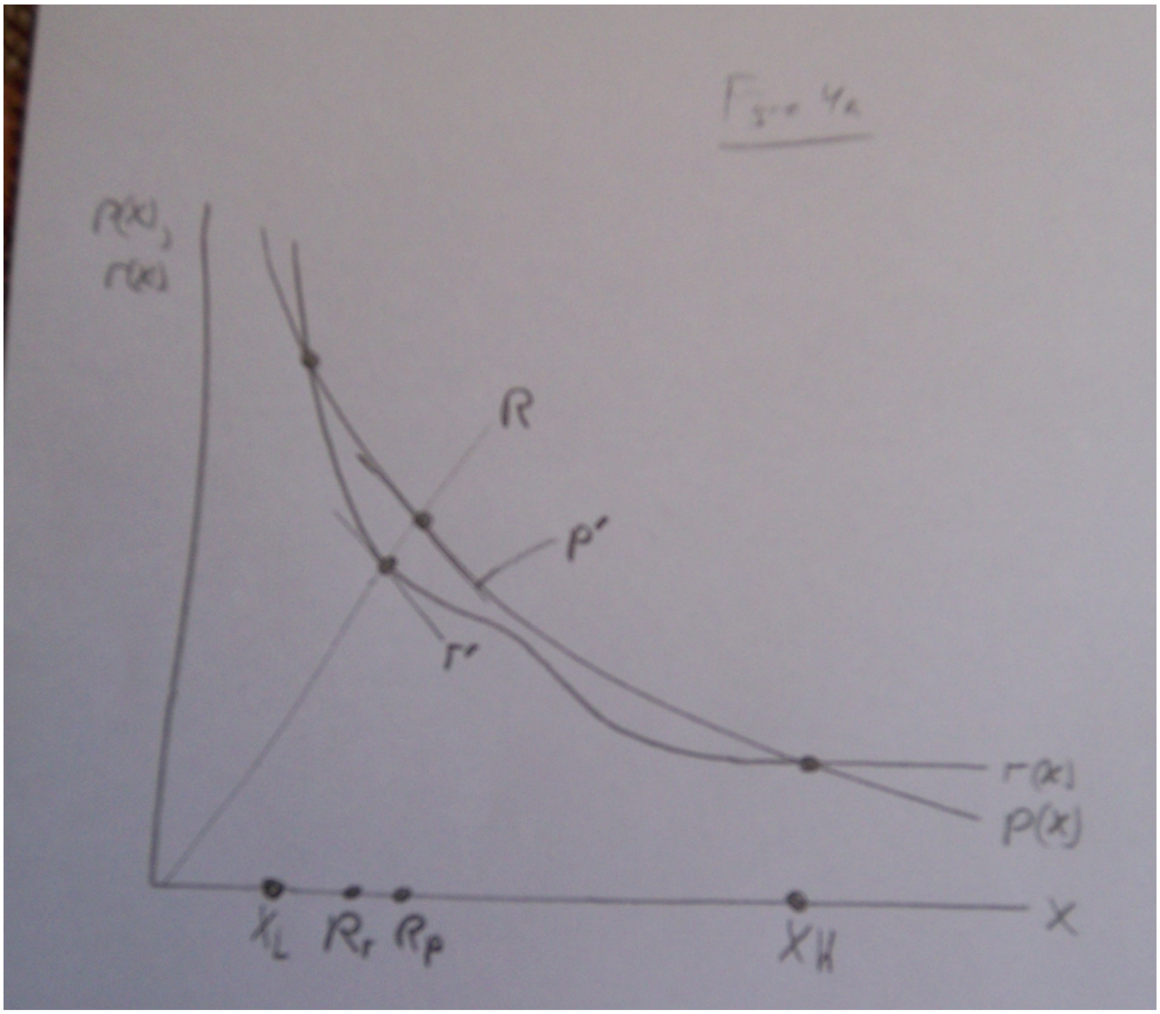


FIGURE 7. Proving Lemma 2

point of $p(x)$ by the same fraction along a ray connecting it to the origin. Choose the transformation equivalent to moving along the ray R just described so that $\hat{p}(x)$ and $r(x)$ are tangent at R_r. ...xxx ran out of time.

Take the rays out of the origin over the relevant range. It's possible to find a ray passing through both functions where the slopes of the functions are the same. Get the outer function and shrink it back along the ray and you end up with a tangency. (This uses property (1).) You can then apply condition (2) to reach a contradiction. xxx □

Corollary to Lemma 2. *If $p(x)$ and $r(x)$ are tangent and $p(x)$ is more convex, then $p(x)$ can never fall back below $r(x)$.*

Proposition 3: *Increased convexity makes λ-perfect price discrimination more attractive relative to posted monopoly pricing: λ^* falls with convexity.*

Proof. Consider two demand curves $p_1(z)$ and $p_2(z)$. First, suppose both are differentiable. Let z^* maximize posted monopoly profit for demand curve 1, which means that $p_1(z^*) + z^* p_1(z^*) = 0$.

Define $\hat{p}_2(z) \equiv \alpha p_2(\beta z)$. This new demand curve has maximum profit at z_m such that $\hat{p}_2(z_m) + z_m \hat{p}_2(z_m) = 0$ Restating in terms of p_2, it is then true that $\alpha p_2(z_m) + \alpha\beta z_m p_2(\beta z_m) = 0$. Pick β so that $\beta z_m = z^*$. Since changing the value of α will not affect the value of β, we can choose the value of α so that $\alpha p_2(\beta z_m) = p_1(z^*)$.

Now we have a transformed inverse demand function $\hat{p}_2(z)$ that is merely $p_2(z)$ with the units changed but is maximized at $(z^*, p_1(z^*)$. Moreover, since the $\hat{p}_2(z)$ demand's profit is maximized at z^*, its marginal revenue must be the same at that quantity also, which means (since its price level is $(p_1(z^*))$ that its slope must be the same as that of $p_1(z)$.

This transformation can be done regardless of whether $p_1(z)$ and $p_2(z)$ can be ordered by our convexity partial ordering. For them to be ordered, any choice of α and β must yield the same ordering.

If they can be ordered (but not only if), and (without loss of generality) $p_1(z)$ is more convex than $p_2(z)$, then in the neighborhood of $(z^*, p_1(z^*))$ $p_1(z)$ lies above $p_2(z)$ for every z in the neighborhood.

What about z not in the neighborhood of z^*? We will use Lemma 2 to address that. FINISH THIS. □

The importance of the concavity of the demand curve to the amount of deadweight loss relative to monopoly profit is examined in detail in Anderson & Renault (2003) for the case where $p''(z)z + 2p'(z) \leq 0$, which ensures existence and uniqueness of Cournot equilibrium. They note that the ratio of deadweight loss plus consumer surplus to monopoly profit is one-half for linear demand, which is the property that gives rise to Proposition 1 here, and that if demand is concave (so demand is convex) the ratios of deadweight loss to monopoly profit and of consumer surplus to monopoly profit are greater than one half, the property behind Proposition 5. The bulk of their paper is concerned with using the "ρ-concavity" idea of Caplin & Nalebuff (1991) to characterize the degree of concavity of a demand function and to show how as demand becomes more concave, the ratios of consumer surplus and deadweight loss to Cournot profit (of which monopoly is a special case) fall. Their results could be applied to bargaining versus posted prices.

Concavity of the demand curve is also central to the analysis of the pass-through of increased costs to consumers in posted monopoly pricing, as examined in Bulow & Pfleiderer (1983) and Weyl & Fabinger (2009).

Constant Curvature Demand Functions

We have just looked at which is better for isoperfect price discrimination, concavity and convexity, and what happens as the degree of convexity increases. It is worth thinking about the degree of concavity carefully because we often refer to changes in the elasticity of demand in talking about market power, and elasticity is closely related to curvature. Let us now make that connection, using a narrower class of demand functions, those with constant curvature. To reflect three particular aspects of demand—the location, scale, and shape of the demand curve— let us specify a flexible three- parameter family of inverse demand functions with the convenient property of constant curvature.

Definition. The demand curve has **constant curvature** *if* $p(z) = \alpha + \beta z^{-\gamma}$ *where* $\beta\gamma > 0$.

$p(z) = \alpha + \beta(z+\delta)^{-\gamma}$ also has constant curvature?

The three parameters α, β, and γ index a wide variety of demand specifications and determine the location, scale, and shape. The restriction $\beta\gamma > 0$ (so that $\beta > 0$ if and only if $\gamma > 0$) ensures that the demand curve slopes downward (note tht it rules out $\gamma = 0$, perfectly elastic demand). We will also assume that $\gamma < 1$ so that the marginal revenue curve slopes downward, that marginal cost is constant at c, and that $(\alpha - c)\gamma < 0$ (to rule out equilibrium quantities of zero or infinity).

The curvature of a demand curve is the elasticity of its slope with respect to quantity; that is, the elasticity of $|P'(z)|$ with respect to z. For the specific case considered here,

$$\text{Curvature} \equiv -\frac{zp''(z)}{p'(z)} = -\frac{z(\gamma+1)\gamma\beta z^{-\gamma-2}}{\gamma\beta z^{-\gamma-1}} = 1 + \gamma, \tag{6}$$

which for the specification $p(z) = \alpha + \beta z^{-\gamma}$ is constant with respect to z. In the language of chpater 2 of Robinson (1933) this is the "adjusted concavity" of demand.

The curvature $1+\gamma$ takes the same sign as $p''(z)$ and so $p(z)$ is convex if and only if $\gamma \geq -1$.[8] Comparing two members $p(z)$ and $\widehat{p}(z)$ of the constant-curvature family (and using an obvious notation) the former is "more convex" than the latter if and only if $\gamma \geq \hat{\gamma}$.

The family of constant-curvature demand curves includes many familiar specifications. Setting $\alpha = 0$ and $\gamma > 0$ (and hence $\beta > 0$ so as to meet the requirement $\beta\gamma > 0$) yields a demand specification with constant price elasticity $1/\gamma$. Thus, being "more convex" is closely related to being "more inelastic " for constant-curvature demand curves. Similarly, setting $\gamma = -1$ (and hence $\beta < 0$) and $\alpha > 0$ yields linear demand. Figure 8 shows four examples of constant-curvature demand curves..

Our assumptions guarantee three regularity conditions.

(1) Positive production is socially desirable because $p(0) > c$. If $\gamma > 0$ then $p(0)$ is infinite. If $\gamma < 0$, then our assumption implies $p(0) = \alpha > c$.

(2) The monopolist's profit maximization problem is well-behaved. Straightforward derivations confirm that the monopolist's marginal revenue satisfies

$$p(z) + zp'(z) = \gamma\alpha + (1-\gamma)p(z) \tag{7}$$

which is strictly decreasing in z if and only if $\gamma < 1$. Note that for $\alpha = 0$ this assumption ensures that demand is elastic.

(3) The socially optimal price and quantity are finite. Setting the marginal revenue above equal to the marginal cost c yields an interior solution at which $p = \frac{c-\gamma\alpha}{1-\gamma}$ This yields a finite price if $\gamma < 1$. It yields a finite quantity whenever $\gamma < 0$. If $\gamma \in (0,1)$ it yields a finite quantity if $\gamma < c/\alpha$, which is true for that γ under our assumption that $(\alpha - c)\gamma < 0$ because then $c > \alpha$ and $\gamma < 1 < c/\alpha$.

Given these conditions the welfare-maximizing and monopoly outputs are straightforward.

Lemma 0. *Suppose that the inverse demand curve has constant curvature, so that $p(z) = \alpha + \beta z^{-\gamma}$ for parameters satisfying $\beta\gamma > 0$, and impose the regularity conditions $\gamma < 1$ and $(\alpha - c)\gamma < 0$. The socially efficient quantity z^* and maximum feasible social surplus*

[8] This measure of curvature takes the same form as the classical coefficient of relative risk aversion in Pratt (1964) only applied to an inverse demand function rather than a utility function.

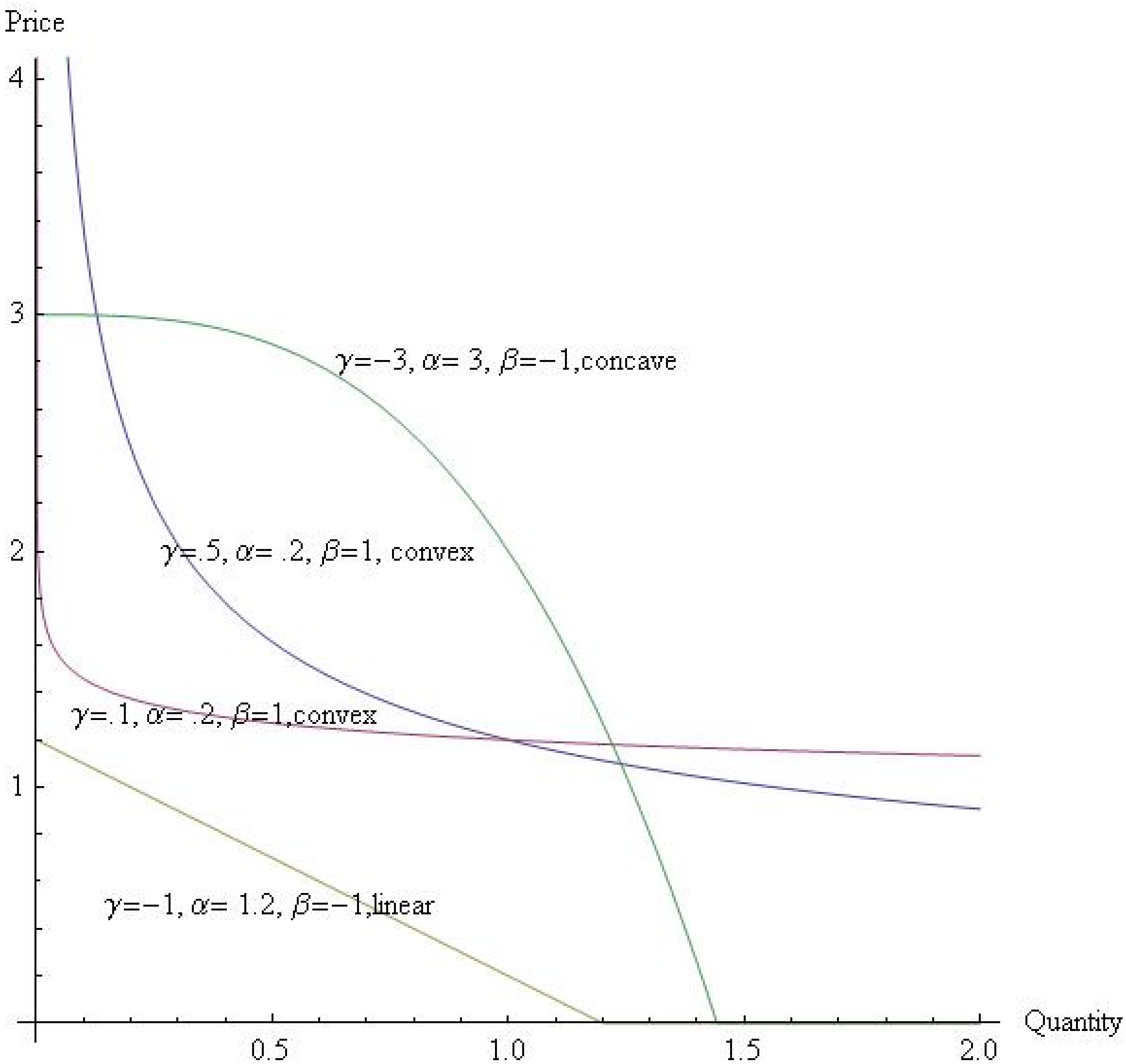


FIGURE 8. Various Constant-Curvature Demand Curves

S^* *satisfy*

$$z^* = \left[\frac{\beta}{c-\alpha}\right]^{1/\gamma} \quad \text{and} \quad S^* = \frac{\gamma}{1-\gamma}\frac{\beta^{1/\gamma}}{(c-\alpha)^{(1-\gamma)/\gamma}}. \tag{8}$$

The monopolist's optimal quantity and profit are $z^m = m(\gamma)z^*$ *and* $\pi^m = m(\gamma)S^*$ *where* $m(\gamma)$ *is the posted-pricing share of social surplus, the* **monopoly power index** *defined by:*

$$m(\gamma) \equiv (1-\gamma)^{1/\gamma}. \tag{9}$$

We should combine these two lemmas, and shorten the exposition perhaps (tho if it is all one proof, readers will be able to skip easily.

Proof. The social surplus from producing z units is

$$S(z) = \int_0^z [p(x) - c]\, dx = \frac{\beta z^{1-\gamma}}{1-\gamma} - (c - \alpha)z. \tag{10}$$

Given that $(\alpha - c)\gamma > 0$ this surplus is maximized at the positive quantity where price reaches marginal cost; that is, at a production level z^* satisfying $p(z^*) = c$. Straightforward calculations confirm that the solution is the expression for z^* given in the statement of the lemma, and plugging back into $S(z)$ yields the expression for S^*.

Turning to the monopolist's problem, the condition $\gamma < 1$ guarantees concavity of the objective function. The monopolist's output solves the usual first-order condition $p(z^m) + z^m P'(z^m) = c$. Using Equation (7) and solving yields

$$z^m = \left[\frac{\beta(1-\gamma)}{c-\alpha}\right]^{1/\gamma} = (1-\gamma)^{1/\gamma}\left[\frac{\beta}{c-\alpha}\right]^{1/\gamma} = m(\gamma)z^* \tag{11}$$

as claimed in the lemma. Calculating the corresponding monopoly price,

$$p(z^m) = \alpha + \beta[m(\gamma)z^*]^{-\gamma} = \alpha + \frac{c-\alpha}{1-\gamma} \quad \Rightarrow \quad p(z^m) - c = \frac{\gamma(c-\alpha)}{1-\gamma}. \tag{12}$$

Calculating the monopolist's profit yields $z^m[p(z^m) - c] = m(\gamma)S^*$ as claimed.
■

Hence if the monopolist commits to monopoly pricing then he supplies a fraction $m(\gamma)$ of the socially optimal output and extracts a fraction $m(\gamma)$ of the maximum feasible social surplus. This fraction depends only on the curvature of demand; it is independent of the location and scale parameters α and β. In essence, it is an index of the monopolist's market power. A special case is $\gamma = -1$, yielding linear demand, the boundary between concave and convex. For this special case, the index satisfies $m(\gamma) = 1/2$, leading back to the familiar insight that a monopolist is able to extract half of the feasible social surplus. More generally, changes in curvature change the monopolist's market power index, as described in the next lemma.

Lemma 1. *If the curvature parameter satisfies $\gamma < 1$ then the "monopoly power index" $m(\gamma)$ satisfies $0 < m(\gamma) < 1$. It is decreasing in γ, satisfying $\lim_{\gamma\to 1} m(\gamma) = 0$ and $\lim_{\gamma\to-\infty} m(\gamma) = 1$.*

Proof. Recall that the restriction $\gamma < 1$ has been imposed to ensure that marginal revenue is downward sloping, and hence $m(\gamma) > 0$. A moment's inspection confirms that $m(\gamma) < 1$. It remains, therefore, to consider the response of $m(\gamma)$ to changes in γ. Now,

$$\log m(\gamma) = \frac{\log(1-\gamma)}{\gamma} \quad \Rightarrow \quad \frac{\partial \log m(\gamma)}{\partial \log \gamma} = -\frac{1}{\gamma^2}\left[\frac{\gamma}{1-\gamma} + \log(1-\gamma)\right]. \tag{13}$$

Hence $m(\gamma)$ is decreasing in γ if and only if the bracketed term is positive. Now,

$$\frac{\partial}{\partial \gamma}\left[\frac{\gamma}{1-\gamma} + \log(1-\gamma)\right] = \frac{\gamma}{(1-\gamma)^2}, \tag{14}$$

and there is a unique stationary point at $\gamma = 0$. Taking the second derivative, we obtain

$$\frac{\partial^2}{\partial \gamma^2}\left[\frac{\gamma}{1-\gamma} + \log(1-\gamma)\right] = \frac{1+\gamma}{(1-\gamma)^3}, \tag{15}$$

which is positive for $\gamma = 0$. Hence the bracketed term is minimized at $\gamma = 0$, at which point it vanishes, so that $m'(0) = 0$. This implies that $m'(\gamma) < 0$ for all $\gamma \neq 0$. Thus a monopolist's power to extract surplus is decreasing in the convexity of the demand function.

Turning to the final two claims of the lemma, note that $\lim_{\gamma\to 1} m(\gamma) = 0$ by inspection, and $\lim_{\gamma\to -\infty} m(\gamma) = 1$ can be obtained by applying l'Hôpital's rule to $\log m(\gamma)$. ■

This lemma tells us that the monopolist's monopoly power (in the sense of his ability to extract social surplus) falls as the curvature (that is, the convexity) of the inverse demand curve increases. At the same time, the increase in convexity provokes an increase in the profit margin. Straightforward calculations confirm that

$$p(z^m) = c + \frac{\gamma(c-\alpha)}{1-\gamma}, \tag{16}$$

which is increasing in γ whenever $c > \alpha$ and hence (given that $(\alpha - c)\gamma < 0$) whenever $\gamma > 0$. This means that for the special case of constant elasticity (so that $\alpha = 0$) a move toward relatively inelastic demand (that is, an increase in γ) increases price-cost margins whilst reducing the market power of the monopolist.

Lemmas 3 and 4 go some way toward characterizing the degree of market power wielded by a monopolist who commits to a uniform price. We now use

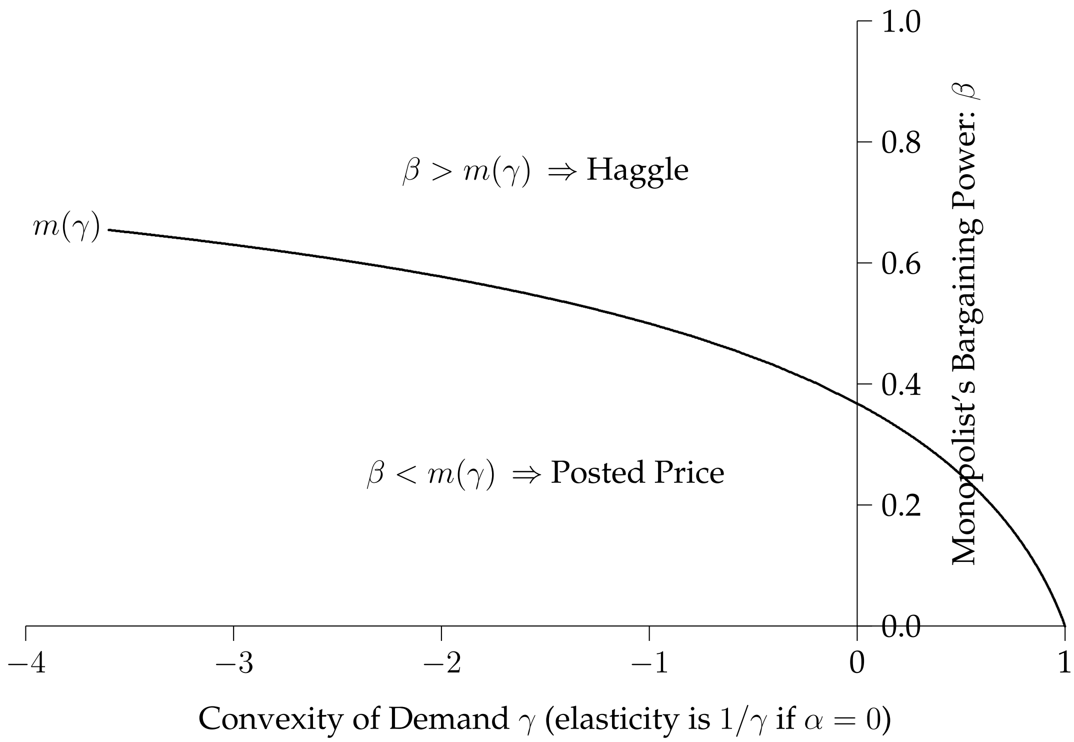


FIGURE 9. A Monopolist's Decision: Price Discrimination vs. Posted Price

these simple results to investigate how the shape of demand influences the monopolist's choice of sales strategy.

When demand has constant curvature, the profit from monopoly pricing is $\pi^m = m(\gamma)S^*$ where S^* is the maximum feasible surplus and $m(\gamma)$ is the index of market power. In contrast, the profit from IPD is $\pi^* = .5S^*$. Clearly, the choice of sales strategy is determined by a comparison of (i) the monopolist's bargaining strength λ and (ii) his monopoly power $m(\gamma)$. Figure 9 illustrates, while the following Proposition 4, a direct result of Lemma 4, states it.

Proposition 4: *The more convex is the curvature of inverse demand (the bigger is γ), the lower can be the seller's bargaining power λ for him to prefer λ-perfect price discrimination to posted monopoly pricing.*

One special case of convex demand is the constant-elasticity demand curve. For such demand, the degree of bargaining power required for perfect price discrimination to be preferred falls as demand becomes less elastic.

Corollary. *When demand has constant elasticity ($\alpha = 0$), the monopolist's desire to adopt λ-isoperfect price discrimination grows as the elasticity of demand falls.*

This is perhaps surprising. We think of inelastic demand as being good for monopoly pricing— and indeed it is. It is even better, however, for isoperfect price discrimination.

6. Dispersion of Tastes and Endogenous Product Variety

Since monopoly pricing captures the entire surplus in the extreme case of homogeneous buyers, all with the same reservation price, and IPD captures only half in that situation, a first guess would be that increased dispersion of consumer values increases the attractiveness of IPD. We will see that this is generally true, but not always.

First, consider the demand curve with constant elasticity $1/\gamma$ with which we concluded the last section of the paper:

$$p(z) = \beta z^{-\gamma} \tag{17}$$

This is the demand that results if consumer valuations θ follow a Pareto distribution with density and cumulative distribution as follows

citation?, derivation?

$$f(\theta) = \frac{(1/\gamma)\beta^{(1/\gamma)}}{\theta^{1/\gamma+1}}, \qquad F(\theta) = 1 - \left(\frac{\beta}{\theta}\right)^{1/\gamma}, \quad E(\theta) = \frac{\beta}{1-\gamma} \tag{18}$$

If the curvature parameter γ increases, the result is a mean-preserving rotation of F, which is a mean-preserving spread of the density $f(\theta)$. Thus, we have a new interpretation of the convexity result from the last corollary:

The last figure should have a couple of different pareto densities for illustration of how elasticity changes it.

Corollary. *An increase in the heterogeneity of valuations having a Pareto distribution favors λ-isoperfect price discrimination over posted monopoly pricing.*

On the other hand, consider demand for which consumer valuations are drawn from a uniform distribution with mean μ and heterogeneity range s:

$$\theta \sim U\left[\mu - \frac{s}{2}, \mu + \frac{s}{2}\right] \tag{19}$$

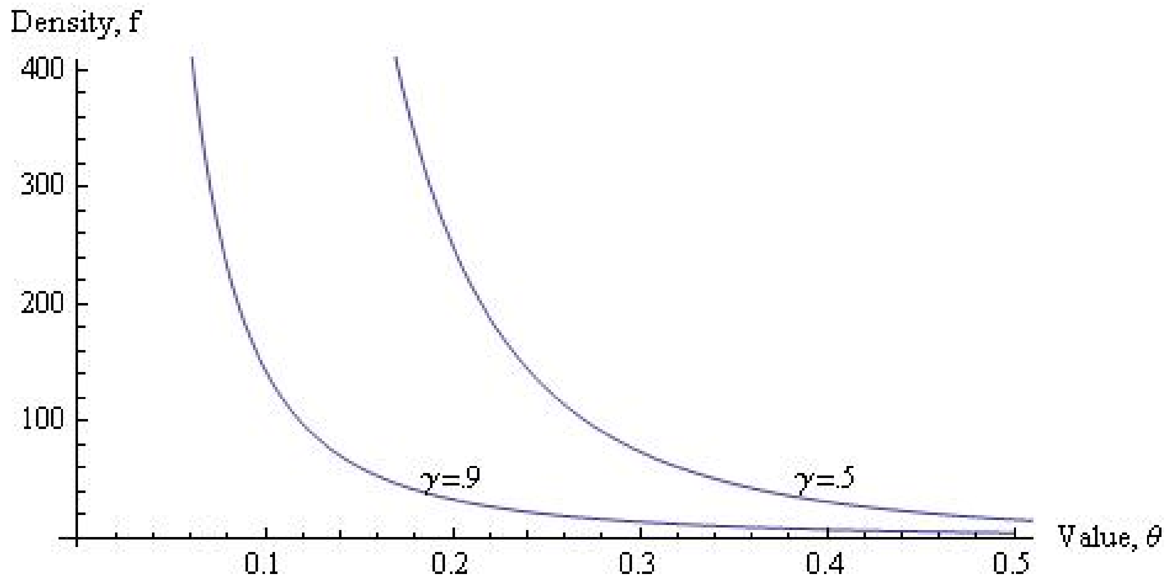


FIGURE 10. Consumer Valuations with a Pareto Density

which corresponds to the inverse demand curve

$$\begin{aligned} p(z) &= \left(\mu + \tfrac{s}{2}\right) - sz \quad && if\ q \le 1 \\ &= 0 && if\ q > 1 \end{aligned} \tag{20}$$

This is not quite the standard linear demand curve, because maximum quantity demanded can (but need not) occur at a strictly positive price, as shown in Figure 11b and Figure 12.

Let us look at the special case of constant marginal cost with $\mu - s/2 < c$, so that in a competitive market some consumers would go unserved because their valuations are below marginal cost. Figure 11 shows two possible scenarios for this: panel (b) with $\mu > c$ and panel (d) with $\mu < c$. Each scenario shows how an increase in the disperson of valuations (an increase in s) would cause the demand curve to rotate around the point $(q = \mu, p = p(\mu))$.

Notice that all of the demand curves in parts (b) and (d) of the figure are linear in the relevant range above marginal cost. From Proposition 1, we know that the seller is therefore indifferent between balanced isoperfect price discrimination and posted monopoly pricing for all of them, despite the increased dispersion of

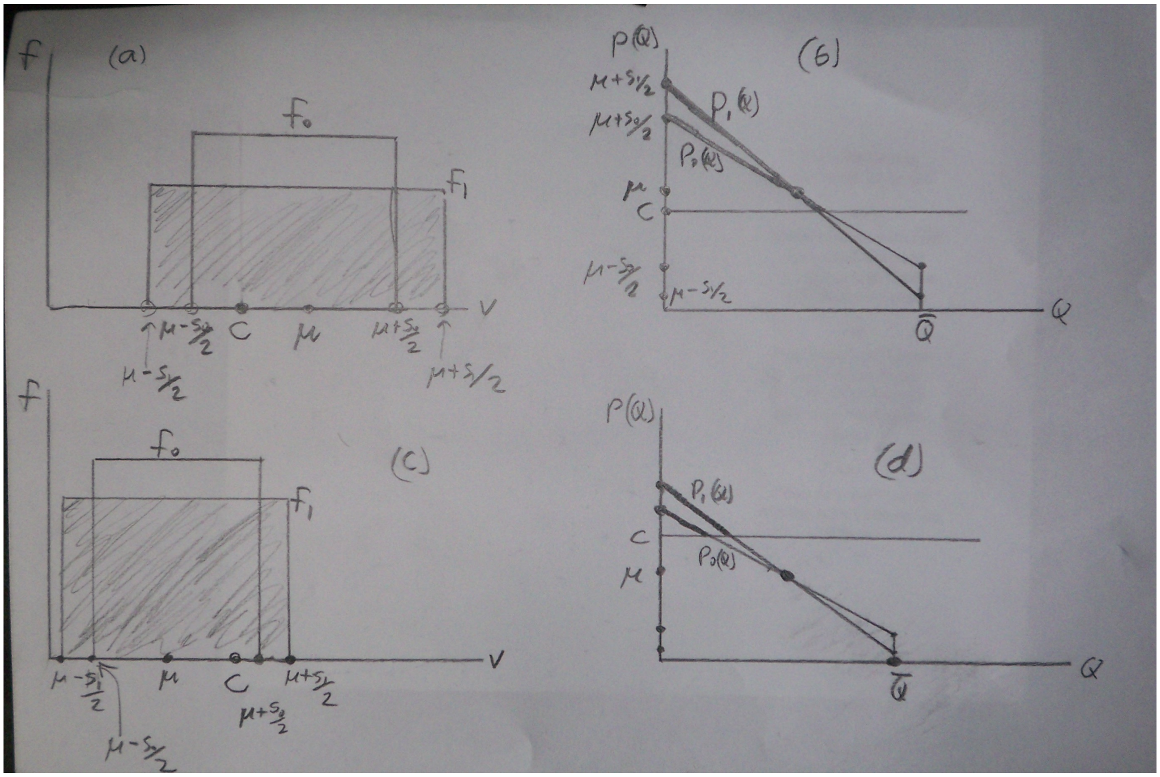


FIGURE 11. Consumer Valuations and Demand Curves

consumer valuations. Thus, increased dispersion does not necessarily give an advantage to price discrimination.[9]

Figure (d) and Figure 12b go a step further. There, the elasticity of demand increases for every relevant q, yet isoperfect price discrimination does not gain any advantage over posted monopoly pricing. To see that the elasticity increases, note that the elasticity is $(dz/dp)(p/z)$. For any value z, the demand curve p_1 has a gentler slope dp/dz and thus a larger value of dz/dp. For given z, it also has a higher price p. Thus, the elasticity is strictly greater for p_1 than for p_2.

Finally, in Figure 12c), where marginal cost is even lower, demand curve $p_2(z)$ has become concave by our definition. Thus, by Proposition 1, posted monopoly pricing is strictly more profitable than isoperfect price discrimination. Increased heterogeneity has reduced the value of price discrimination. Proposition 5 states this result.

[9]In the Demand Changes diagrams, put thick black lines on the dmeand curves above the MC line. All the lines need to be blacker. Add case (d), where the MC falls so lo that both demand curves completely satisfy the market at P=MC. We will add a big discussion of that case.

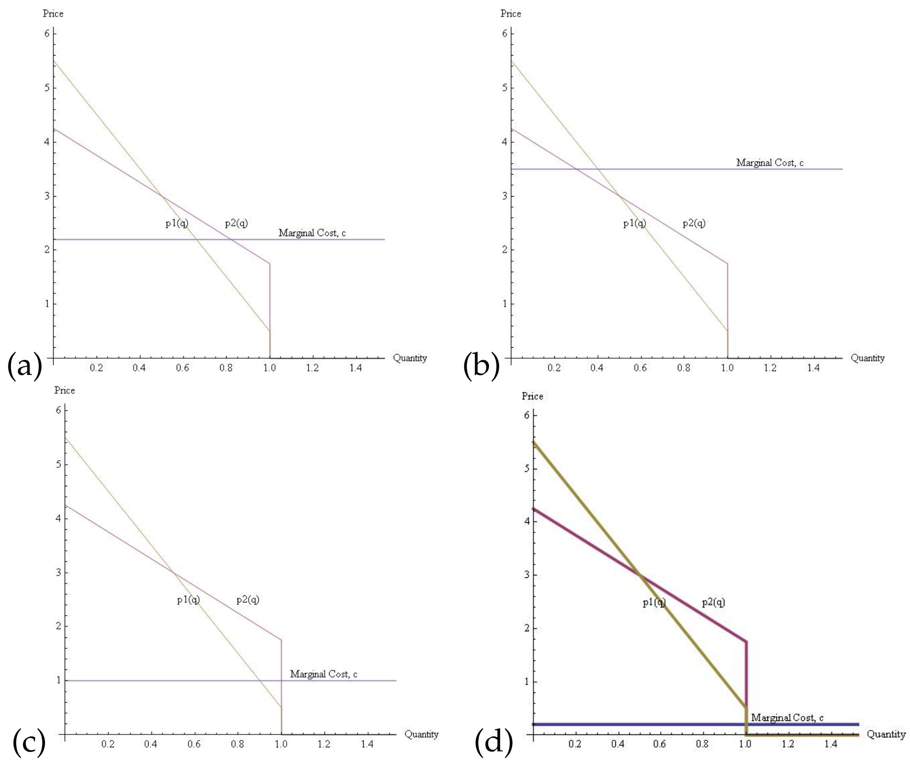


FIGURE 12. Demand Changes above Various Levels of Marginal Cost

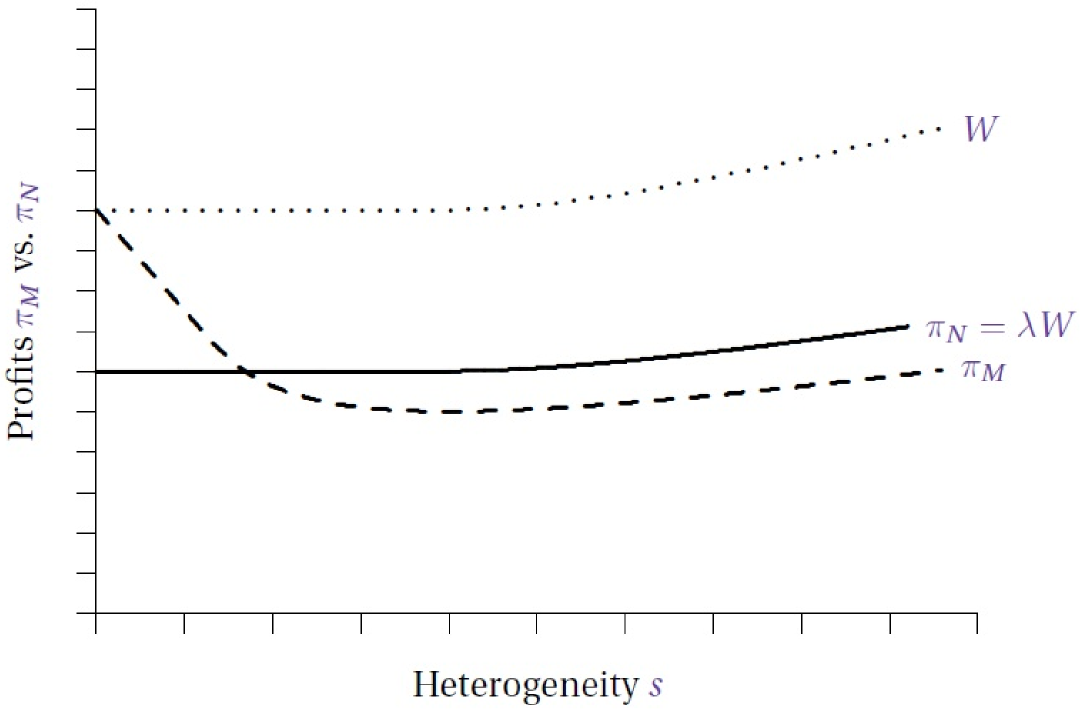
Proposition 5: *Increased dispersion of consumer types and increased elasticity of demand for every quantity can make isoperfect price discrimination either more profitable than posted monopoly pricing, or less.*

One reason why our result in Figure 12b — where isoperfect price discrimination and posted pricing yielded equal profits— is that increased heterogeneity does increase the profits from price discrimination. What we may miss, however, is that it also increases the profitability of posted monopoly pricing in that situation, where $\mu < c$.

Proposition 6. *If $\mu < c$ in the heterogeneous-tastes model, then profits from isoperfect price discrimination and posted monopoly pricing are equal and increasing in the heterogeneity parameter s.*

We will probably want one big proposition for this section, saying what happens as MC falls, or maybe for constnat MC as heterogeneity rises. David's IU 2011 slides have good stuff for this.

Proof. Under neither pricing policy will the monopolist produce quantities such that the lowest value of any purchaser, $p(q)$, is less than marginal cost c, since then some sales would have to be below marginal cost. Thus, only quantities where $p(q) \geq c$ are relevant to its pricing. As shown in Figure 12b), both demand curves p_1



(a)

FIGURE 13. Heterogeneity and Profits for Linear Demand

and p_2, where s is greater for p_2, are linear for $p > p(q) = c$. Suppose we artificially extend those demand curves linearly all the way down to $p(q) = 0$ for some q before we analyze optimal pricing. This will not change the optimal pricing policy, since those quantities are irrelevant to it. Our new demand curves, however, are completely linear, so Proposition 1 tells us that for both of them, the profits from isoperfect price discrimination and posted monopoly pricing are equal.

Next, observe that for any quantity below q_0 such that $p_1(q_0) = c$ it is true that $p_2(q) > p_1(q)$. In particular, if we denote by q_m demand curve 1's posted monopoly output,$p_2(q_m) > p_1(q_m)$. Posted monopoly profits are higher with the greater s of demand curve 2 even at q_m, and would be even higher if we optimized by choosing demand curve 2's monopoly output. ■

It seems, then, that product variety and increased consumer information to increase dispersion should be more attractive to a posted-pricing monopolist than to one that knows consumer values and is forced into isoperfect price discrimination.

Here we will talk about endogenous product variety and the complementarity (or opposite) of IPD and educating the consumer to make tastes

Perhaps we should add discussion of when $f(\theta)$ is monotonic, generating a sort of S-shaped demand curve(you know what I mean!). That demand curve is neither concave nor convex, but it's very realistic. We could give a numerical solution to an examlpe, using Mathetmatica, and address generally how if the seller has enough info to divide the consumers into two groups, he would use IPD for the high-value demanders and posted pricing for the lower-value, which would, however, include some above-average consuemrs on the convex part oft eh demand curve. I think in some cases, in fact, even with the option to split the groups, the seller would use two poasted prices instead of IPD plus a posted price.

Possibly split off this section to another paper.

Selling Multiple Products

So far our monopolist has been selling one product, though we have let him choose the features of that product. What if he can sell several products? This is classic second-degree price discrimination as in Mussa & Rosen (1978). Does the ability to sell multiple products make monopoly pricing more attractive relative to isoperfect price discrimination? On the one hand, the multiple products allow the monopolist to differentiate among consumers even using posted prices. On the other hand, the multiple products increase the total market surplus, and thus increase the profitability of isoperfect price discrimination.

Proposition 7:*A monopolist's ability to sell multiple products instead of one does not affect the profitability of isoperfect price discrimination relative to monopoly pricing.*

The Proposition above of course is stated in the context of our basic model, with linear demand and constant marginal cost. We must modify the basic model, however, to say what it means to sell multiple products. Let us assume that the linear demand curve and constant marginal cost for each product arise as follows. The monopolist may sell n goods. Good i has quality $q = q_i$ and constant marginal cost $c(q)$ with $c'(q) > 0$ and $c\prime\prime(q) \geq 0$ (which imply that the marginal cost of one unit increases with the unit's quality, and increases at the same rate or faster than quality). We will order the goods from lowest to highest quality, so q_1 is the lowest possible quality. A unit mass of consumers have types θ distributed according to $F(\theta)$, where a consumer's utility from buying a good of quality q_i and price p_i is

$$u_i(\theta) = \theta q_i - p_i. \tag{21}$$

This assumption that type and quality multiply each other in the utility function is substantive, as is the assumption that consumers value each increment of quality equally (i.e., θ is constant rather than being any $\theta(q)$).

Let us start by deleting any product quality which is inefficient, for which there will be zero quantity produced in equilibrium, and renumbering to skip those products. Let us then define Quality 0 as having $p_0 = 0$ and $q_0 = 0$.

Suppose the marginal consumer is at $\theta = \theta^*$. The quantity sold will then be $z(p) = (1 - F(\theta^*))$. Let $p(z)$ be the inverse of this function.

If there is a single product with quality q_i, then the marginal consumer is the one with $u_i(\theta) = 0$, so for him $p_i = \theta q_i$.

The monopolist's profit from monopoly pricing is

$$\pi(m) = \Sigma_{i=1}^{n} z_i [p_i - c(q_i)] \tag{22}$$

Let us define θ_i as the type of the lowest-valuing consumer who buys quality i in equilibrium. Thus,

$$p_i - p_{i-1} = \theta_i (q_i - q_{i-1}). \tag{23}$$

In particular, using our definition of Quality 0,

$$\theta_1 = \frac{p_1}{q_1}. \tag{24}$$

Consumers of Type 0 don't buy anything in equilibrium. Consumers of Types 1 through n would buy Quality 1 if it was the only quality available. They have mass $q_1 = 1 - F(\theta_1)$, which equals $q_1 = \Sigma_{j=1}^{n} z_j$.

$$\theta_1 = p\left(\Sigma_{j=1}^{n} z_j\right) \tag{25}$$

Therefore,

$$p_1 = q_1 p\left(\Sigma_{j=1}^{n} z_j\right). \tag{26}$$

It will be true that

$$p_2 - p_1 = (q_2 - q_1) p\left(\Sigma_{j=2}^{n} z_j\right). \tag{27}$$

We can say generally that

$$p_{i+1} - p_i = (q_{i+1} - q_i) p\left(\Sigma_{j=i+1}^{n} z_j\right). \tag{28}$$

Following the idea in Johnson & Myatt (2006), let us now look at the n products in a different way, as a single product with quality q_1 and $(n-1)$ upgrades from quality q_i to quality q_{i+1}. The price of the basic product with quality q_1 is p_1, and it is sold to $\overline{z}_1 = \Sigma_{j=1}^{n} z_j$ consumers— to every consumer who buys at all. The price of an upgrade to quality q_2 is $\overline{p}_2 = p_2 - p_1$, it yields an increment $\overline{q}_2 = q_2 - q_1$ in quality, and it is sold to $\overline{z}_2 = \Sigma_{j=2}^{n} z_j$ consumers.

If we lump the basic product and the upgrades together, we have a total of n products, where product i has price $\overline{p}_i = p_i - p_{i-1}$, yields an increment $\overline{q}_i = q_i - q_{i-1}$ in quality, and is sold to $\overline{z}_i = \Sigma_{j=i}^{n} z_j$ consumers.

We can rewrite equation (28) in terms of the new products. It becomes

$$\overline{p}_i = \overline{q}_i p\left(\overline{z}_i\right). \tag{29}$$

We can also rewrite the profit equation (22) in terms of the new products, if we define the marginal cost of a product as $\overline{c}_i = c_i - c_{i-1}$. From our n new products, the monopolist earns

$$\begin{aligned}
\pi(m) &= \Sigma_{i=1}^{n} \overline{z}_i[\overline{p}_i - \overline{c}_i] \\
&= \Sigma_{i=1}^{n} \overline{z}_i[\overline{q}_i p\left(\overline{z}_i\right) - \overline{c}_i] \\
&= \Sigma_{i=1}^{n} \overline{q}_i \overline{z}_i[p\left(\overline{z}_i\right) - \tfrac{\overline{c}_i}{\overline{q}_i}]
\end{aligned} \tag{30}$$

Finally, we can think about the monopolist's optimization problem. Expression (30) can be maximized by choice of the $\overline{z}_i$ very simply. It is the sum of n MR=MC terms, but in term i only $\overline{z}_i$ appears, not any other of the new products' quantities. The only relationship that the different quantities bear to each other is that it must be that $\overline{z}_{i+1} \leq \overline{z}_i$ because the quantity sold of an upgrade cannot be greater than that of the product which is upgraded. That will be a nonbinding constraint in the maximization problem here, because the first-order conditions are, for $i = 1, ldotsn$,

$$p\left(\overline{z}_i\right) + z_i p'\left(\overline{z}_i\right) = \frac{\overline{c}_i}{\overline{q}_i} \tag{31}$$

If $c\prime\prime(q) \geq 0$, as we have assumed, then the right- hand-side of equation (31) is nondecreasing in i, and so the left-hand-side will be nondecreasing in i and the z_i's that solve it will be nonincreasing, which is what we need for the constraint to be nonbinding.

What does this tell us about the profit from monopoly pricing? It says that we can view it as the profit from the monopoly pricing of n independent products, each with constant marginal cost and linear demand function $p(\cdot)$, so for each of the products the monopolist will acquire half of the surplus. But from Proposition 1 we know this is the same profit as a balanced isoperfect price discriminator would get. This proves Proposition 14 above.

8. Further Applications of the Idea of Isoperfect Price Discrimination

One of the ideas behind this paper is that final consumers have market power, regardless of their size. If one person's demand disappears, it is not replaced by somebody else's, unlike supply in a world of perfect competition. Consequently, every transaction is at least monopsonistic, and if the seller has market power it is a bilateral monopoly. One-on-one interaction results in something more like bargaining than like monopoly pricing.

We have focussed on the profitability of posted pricing versus price discrimination, but the underlying idea has a number of applications even by itself.

In law-and economics, a number of authors, including Goldberg (1984), Cooter and Eisenberg (1985), and Scott (1990), have looked at the "lost volume problem." In the paradigmatic case of *Neri v. Retail Marine Corp*, 285 N.E.2d 311, 314 n.2 (N.Y. 1972), Retail Marine agreed to sell Neri a boat. Neri repudiated the contract. Retail Marine then sold the boat to someone else at the same price. Neri argued in court that Retail Marine had suffered no loss, since it had succeeded inselling the boat. The court awarded damages to Retail Marine of $2,579 in lost profit, however, on account of the lost sales volume. That makes sense under our reasoning: Neri was a unique buyer, and when he took his demand away from the market, it was not replaced.

In industrial organization, many problems involve the interaction of wholesalers and retailers. When a monopolist or duopolist wholesaler faces many retailers who are monopolies in their own markets, how should we model this? Commonly, buyers with small market shares are assumed to have no market power, but this is questionable, as Frank Mathewson and Ralph Winter note in saying that "The theoretical explanation of why it is empirically reasonable to impute zero-monopsony power to buyers with small market shares is an open issue not explored here." (Mathewson & Winter p. 1058). In their model, the game is set up so that wholesalers make take-it-or-leave-it offers. I suggest that although in particular contexts this may be appropriate for analytic convenience, it is not realistic. Robert Bork takes the view they are criticizing, saying, "The retailer has alternative suppliers. Standard [the wholesaler] has no alternative outlet with which to reach customers in that town."[10]

[10] xxxx (page number?)

Another commonplace in industrial organization is the possibility of secret discounting that undermines profits in cartels. The standard reasoning is that this allows members of the cartel to compete with each other for customers. As a result, the law should encourage secret discounting. American antitrust law's historical hostility to secret rebates is inconsistent with this, but that can be attributed to either its political basis (protecting businesses that cannot get the rebates) or to lack of understanding of economics.

It is quite true that discounting undermines cartels, but the present paper suggests another reason why it hurts cartels and helps consumers: bargaining price discrimination. Suppose the cartel could allocate customers by territory, so that price competition among members was not a threat. Discounting would still hurt cartel profits, and would still be tempting because of bargaining with individual customers.

The idea of bargaining price discrimination has implications for the desirability of unionization. An argument sometimes made is that the employer is large, and has market power, whereas the worker is small and has no market power. As a result, unionization is helpful to turn simple monopsony into bilateral monopoly.[11]

We have suggested that size is not what is important. Moreover, each worker does have market power, since he is the sole provider of his labor, whereas it is rare for an employer to have a monopsony. Hence, any justification for unionization must rely on other reasons why the worker's bargaining position is weak.

Finally, a basic dichotomy in international trade models is between large countries, which are big enough to influence world prices by their tariff levels, and small countries, which are not. [12] A small country cannot benefit from imposing a tariff, because it cannot cause the terms of trade to change in its favor, but for a large country there is a positive optimal tariff.[13] If tariffs are the result of bargaining between large and small countries, however, and if resale can be prevented, then the situation is one of bargaining price discrimination. The small country's demand is irreplaceable, and it can bargain with the large country for a more favorable price. Pakistan, for example, might be a very minor consumer of coffee,

[11]xxx Find a cite for this.

[12]xxx Find a cite for this.

[13]Optimality is here considered only with respect to the country's interest. From the point of view of world surplus, tariffs are always bad. For one country, however, they can be good, just as for one firm, acting as a monopolist rather than as a price-taker can be good.

but it could nonetheless negotiate with Brazil for a favorable coffee price. This is interesting because the small country has no market power against the world as a whole, but it does against one other large country.

One thing we have not discussed is transaction costs. Monopoly pricing has lower transaction costs, an obvious reason for the seller to prefer it.

8. Concluding Remarks

The idea that consumers have market power in one-on-one bargaining is powerful, and we have talked here about a confusingly large number of implications. Here are a few thoughts that the overwhelmed reader may take away:

1. Monopolists should often be glad, not unhappy, that transaction costs forbid them from engaging in perfect price discrimination.

2. The ability of a seller to extract profits is heavily influenced by the ability to commit. It is not enough to be the only supplier of a product, or to face a downward sloping market demand curve. This has been obvious in bargaining models, but it is true in what are usually considered old-fashioned monopoly models.

3. Being small is not the same as being powerless. An atomistic consumer still has market power unless demand is perfectly elastic. He is the only consumer with that particular level of demand—or at least one of a limited number (maybe demand is elastic over an interval). His problem under a monopoly arises because of transaction costs: he is too small to spread a fixed cost.

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