Game Theory in Finance

Eric Rasmusen

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Abstract

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2000: Eric Rasmusen, Professor of Business Economics and Public Policy and Sanjay Subhedar Faculty Fellow, Indiana University, Kelley School of Business, BU 456, 1309 E 10th Street, Bloomington, Indiana, 47405-1701. Office: (812) 855-9219. Fax: 812-855-3354. Erasmuse@indiana.edu. Php.indiana.edu/∼erasmuse.

Game theory is a modelling approach which drops perfect competition's assumption that individuals are price-takers and instead requires them to behave strategically, taking into account that their actions will alter the behaviour of the rest of the market. This may be in a context in which two players consciously choose actions that affect each other, as in duopoly, but in finance it is more common for one player to try to manipulate the behaviour of competing players on the other side of the market. In either case, game theory addresses strategic behaviour by defining the players in the game, the payoff functions they are maximizing, and the strategies available to them. It is crucial to delineate carefully the order of actions and the information available to each player: precommitment and information transmission are the two pillars of modern game theory.

Game theory's most important contribution to finance is a very old one: the theory of expected utility, which was detailed in the second edition of Von Neumann and Morgenstern's *Theory of Games and Economic Behavior* in 1947. Putting that aside, game theory has been most useful in the context of asymmetric information, which has increased in research importance as returns have diminished in the economics of uncertainty. Finance, perhaps even more than other subjects, is amenable to game theory's approach. In financial markets informational advantages matter, events are cleanly defined, and the important participants are experienced players with enough at stake to justify careful thought. Thus, the stylized models and sophisticated rationality of game theory may apply better to markets for corporations than for cantaloupes.

This article will attempt no more than to convey the flavour of game theory in finance. For particular techniques, see Kreps (1990) or Rasmusen (1989); for references, see Harris and Raviv (forthcoming). Rather than survey the literature, I will here convey its flavour using three typical models, of signalling, commitment, and incentive design.

Example 1. Signalling in Tender Offers: Why Do Tender Offers Sometimes Fail? A major use of game theory is to formalize intuition, obtain a combination of intuitive and counterintuitive results, and then refine the intuition by modifying the model until the results become realistic and their origins understood. A sequence of models of tender offers illustrates this nicely. If a bidder who can increase a target firm's value by z makes a conditional tender offer that is $x < z$ above the current stock price, no shareholders will sell, even though they would jointly benefit. The shareholders are in a prisoner's dilemma: it is better (by $z - x$) to be a holdout than a tenderer if the offer succeeds and no worse if it fails, so every shareholder holds out (Grossman and Hart [1980]). By this argument, tender offers should never occur, but Shleifer and Vishny (1986) point out that if the bidder begins with a stake of α in the company, tender offers can be profitable. The bidder can profit on his original shares even if he offers $x > z$ and loses on the tendered shares.

Hirshleifer and Titman (1990) explain why offers succeed sometimes, but not always. Nature chooses the bidder's synergy value—his "type"— to be $z \in (0, \overline{z}]$. The bidder offers a premium of x for each of proportion ω of the shares, and each of a continuum of shareholders decides whether to accept or reject the offer. If over $(.5 - \alpha)$ accept, the payoffs are x for those that accept and z for those that refuse; otherwise, all payoffs are zero.

Two kinds of equilibria are possible. One kind is a "separating equilibrium" in "mixed strategies": each type of bidder behaves differently and the shareholders randomize their behaviour. The bidder offers $x = z$ for $\omega = .5$ of the shares, and the shareholders randomize their acceptances so that with probability $(x/\overline{z})^{\omega/\alpha}$ the offer succeeds. The high-z bidder will not offer a lower x because the offer would more likely fail and he would lose the potential gain on his initial α shares. A second kind of equilibrium, less plausible here, is a "pooling equilibrium," in which different types of bidders behave the same. In one of the pooling equilibria, $x = 0$ for any z, and offers always fail because any positive offer would be rejected—the shareholders would all hold out under the "out-of-equilibrium belief" that if $x > 0$, then $z = \overline{z}$. Pooling equilibria are ruled out here by the reasonable out-of-equilibrium belief that a higher bid signals a higher value of z, in which case a low-z

bidder could profitably deviate from the pooling equilibrium by offering a low x and the shareholders would accept his offer.

This is an example of a "signalling" model, with a low premium signalling low synergy. What makes a signal credible is that it be more expensive for one type of player than for another, as the low-premium offer (with its lower probability of success) is for the low-synergy player. Signalling models often have multiple equilibria, and much research effort has been devoted to refining the equilibrium concept to reduce the number of equilibria (see Chapter xxx of Kreps [1990]).

Example 2. Capital Structure as Precommitment: Can High Debt Help Business? Players often undertake actions to restrict their actions or information later in a game. An example is the following capital structure model based on Brander and Lewis (1986). The players are two firms in the same market. In the first move, the firms simultaneously choose debt levels, and in the second they simultaneously choose output levels q_1 and q_2. Nature then chooses the level of a random demand shock z and profits are realized. It is assumed that firm i's profit, $\Pi_i(q_i, q_j, z)$, is decreasing in q_j and increasing in z, and that the *marginal* profit $(\partial\Pi_i/\partial q_i)$ is increasing in the shock z. When z is large, a firm's profits are higher, especially if it has chosen a high output level.

If both firms choose zero debt, this is the Cournot game with uncertainty: the firms trade off the advantage of high output when z is large against the disadvantage when z is small. A firm with heavy debt, however, would go bankrupt if z were low in any case, and its shareholders do not care about the disadvantage of high output in that state, thanks to limited liability. They do benefit from high output when z is high, so heavy debt is an incentive for high output.

A seller in a Cournot duopoly would like to be able to commit to high output, because this induces his rival to choose a lower output. Debt is a form of precommitment. When firm i incurs debt, firm j knows that i's incentive to produce high output has increased, so j will cut back. If both firms incur debt, however, which is the equilibrium here, both of their incentives for high output have increased, and compared to zero debt both outputs are greater and both profits lower.

Unlike Example 1, this is a game of symmetric information, where the focus is on commitment, not information transmission. Each firm deliberately risks bankruptcy to create a conflict of interest between debt and equity that increases its aggressiveness in seeking market share. The outcome is worse for the firms than if they jointly avoid debt, because debt lowers industry profits while helping the firm that uses it as a commitment tool—another example of the prisoner's dilemma.

Example 3. Incentive Design: Why Use Financial Intermediaries? In some models, the players begin with symmetric information, but they know that certain players will later acquire an advantage. In Diamond's 1984 model of financial intermediaries, M risk-neutral investors wish to finance N risk-neutral entrepreneurs. Each entrepreneur has a project that requires 1 in capital and yields Y, where Y is initially unknown to anyone. If $Y < 1$, he honestly cannot repay the investors, but the problem is that only he, not the investors, will observe Y, so they cannot validate his claim that $Y < 1$. They must rely on one of two things to elicit the truth: monitoring or an incentive contract. Under monitoring, each investor pays K to observe Y, which makes it a contractible variable, on which the repayment can be made contingent. Under the incentive contract, the entrepreneur suffers a dissipative punishment $\phi(z)$ if he repays z. The cost of monitoring is MK, while the expected cost of an incentive contract is $E\phi$, so in the absence of an intermediary the incentive contract is preferred if $E\phi < MK$.

The purpose of a financial intermediary is to eliminate redundancy by replacing the M individual monitors with a single central monitor. The intermediary itself requires an incentive contract, at cost $E\phi$, and it must spread this cost over several entrepreneurs to make its existence worthwhile. Otherwise, if $N = 1$, the intermediary incurs a cost of K for monitoring and $E\phi$ for its own incentive, whereas a direct investor-entrepreneur contract would cost only $E\phi$.

The use of this model is to show that (a) an intermediary helps only if there are both many investors and many entrepreneurs, and (b) incentive contracts have economies of scale compared to monitoring. The model is an example of theory-based institutional economics: the institution takes its particular form to avoid information problems by contracting while information is still symmetric.

Each of the three models discussed above is typical of a literature, and other literatures in finance (e.g., executive compensation, market microstructure) also use game theory. The number and intricacy of the models can be daunting, and they have been criticized for the difficulty of empirically verifying the assumptions and for sensitivity to seemingly minor changes in assumptions about what information is available or who moves first. Whether these criticisms apply depends on the model, but they may apply without being truly objectionable. It is unfortunate if important variables are hard to measure, but that does not diminish their importance; rather, case-by-case verification must replace the regression-running that has dominated economists' empirical work. Sensitivity to assumptions is not a drawback but a *contribution* of game theory, pointing out the importance of what were once thought to be insignificant details of the world. Just as marginalism is more than the application of calculus to old problems in economics, so game theory is as important for changing the agenda as for introducing new techniques.

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