$(2(10^n)/(p-1))^{1/(p-1)}$, and if we use $S_m - g(m+1)$ to approximate S, we obtain n place accuracy if $m > (2(10)^n)^{1/p}$.

The case $p = 1$ ignored in Example 2 can be dealt with even though the natural logorithm has not been introduced (see, e.g., [**1**], p. 566).

EXAMPLE 3. Given $\sum_{n=3}^{\infty} n(n^2-7)^{-5/2}$, let $g(x) = -(x^2-7)^{-3/2}/3$. Then if $x > \sqrt{83.3}$, $-g(x) < .5(10)^{-3}$, so that S_{10} approximates S accurately to three decimal places by (1).

In the above example a nice antiderivative g was readily apparent. If this is not the case, comparison techniques can sometimes be used by noting that if $0 < f_1(n) \leqslant f_2(n)$ for $n \geqslant k$ and $E_n^{\,i}$ is the error term for f_i $(i = 1,2)$, then $E_n^{\,1} < E_n^{\,2}$ for $n \geqslant k$. Our next example illustrates this.

EXAMPLE 4. The sum of the first 41 terms, S_{41}, approximates $\sum_{n=2}^{\infty}(n^3+5n-7)^{-4/3}$ accurately to at least 5 decimal places. To see this, note that $(n^3+5n-7)^{-4/3} < n^{-4}$ for $n \geqslant 2$. Thus, if E_n denotes the error term for $\sum_{n=2}^{\infty} n^{-4}$, then by Example 2, $E_m < .5(10)^{-5}$ if $m^{-3}/3 < .5(10)^{-5}$ or $m \geqslant 41$. We thereby obtain 5-place accuracy for the original series.

None of the preceding—examples or theory—requires the concept of the definite integral. However, if the definite and improper integrals have been introduced, the Integral Test is an immediate corollary to Theorem (A). Since $\ln x$ and e^x would then be available, we could consider examples of the following type.

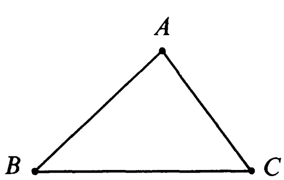
EXAMPLE 5. Inequality (1) assures us that S_4 approximates $\sum_{n=1}^{\infty} e^{-n^2} n$ accurately to 4 places, since if $g(x) = -e^{-x^2}/2$, $-g(m) < .5(10)^{-4}$ when $m \geqslant 4 > 2\sqrt{\ln 10}$.

Reference

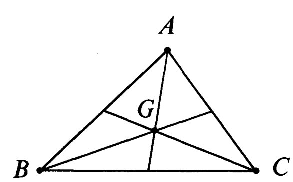
[1] Howard Anton, Calculus with Analytical Geometry, Wiley, New York, 1980.

Triangle Rhyme

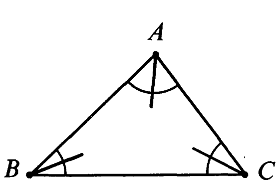
DWIGHT PAINE
Messiah College
Grantham, PA



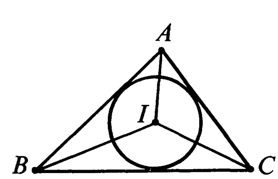
Let's talk about a **triangle**;
We'll call it ABC,



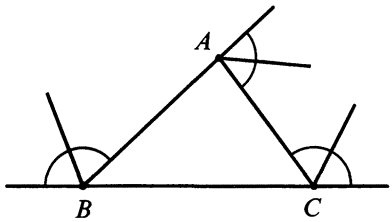
With **medians** converging
To the centroid, labeled G.



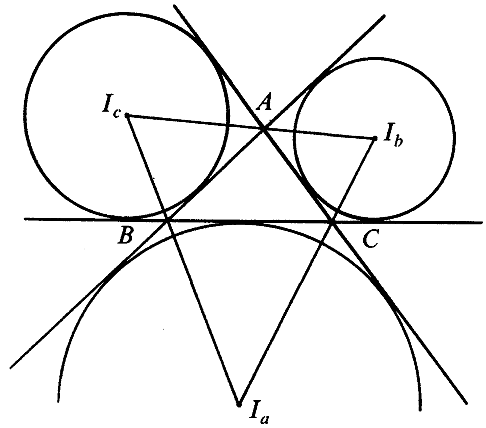
To bisect all the angles
Takes a dreadful steady eye,



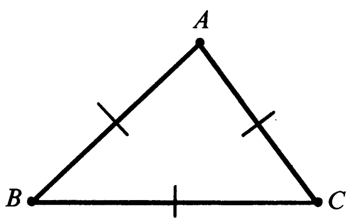
But all the **angle bisectors**
Meet at the **incenter** I.



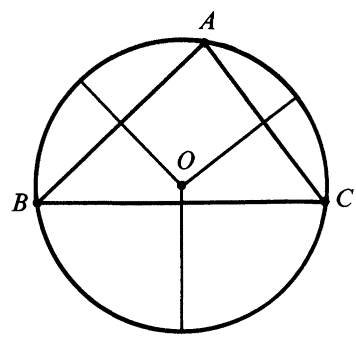
There are **external bisectors**,
Besides the other three,



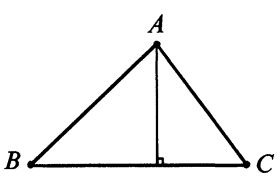
And they meet at the **excenters**:
I_a, I_b, I_c.



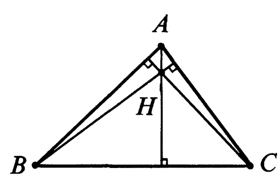
The **perpendicular bisectors**
Through the sides must go,



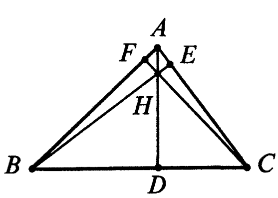
All radiating outward
From the **circumcenter** O.



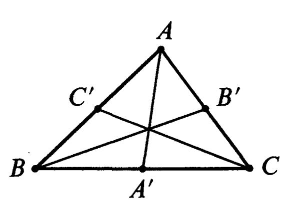
Although the **altitudes** are three,
Remarks my daughter Rachel,



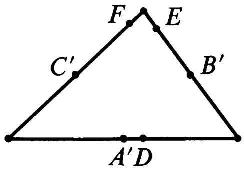
One point'll lie on all of them:
The **orthocenter** H'll.



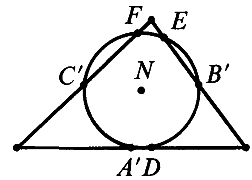
Their feet are often labeled
D, E, F (as in this rhyme),



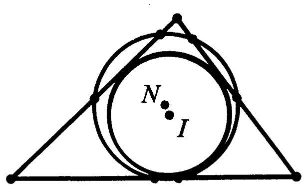
And **medians** too have feet
That we can call A, B, C prime.



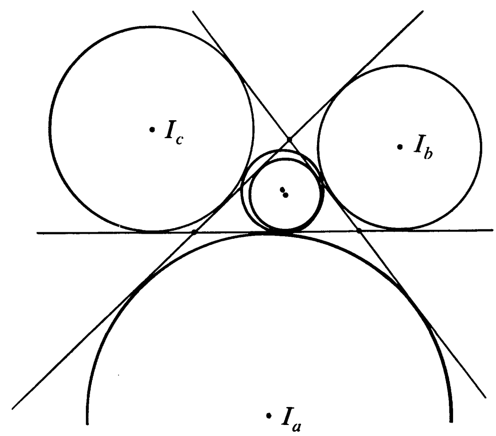
Those feet (I mean all six of them)
Can all be found again



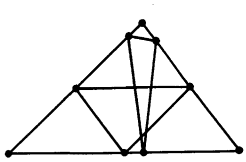
Around the **nine-point circle**
With its nine-point center *N*.



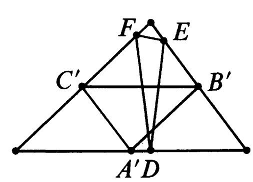
The nine-point circle, by the way,
Does something *hard* to do:



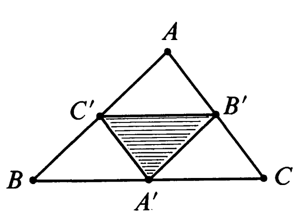
It *touches* the **incircle**
And the three **excircles**, too!



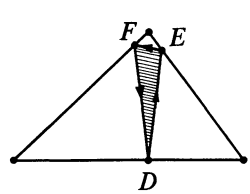
The **medial** and **orthic**
Are triangles inscribed:



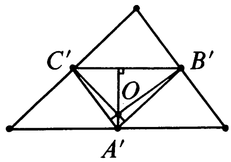
"*ABC*-prime and *DEF*"
Is how they are described.



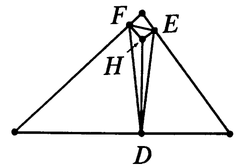
The medial is shaped just like
Its parent *ABC*;



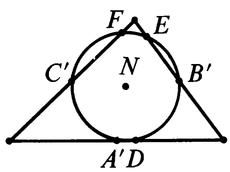
The orthic makes the shortest path
To touch the sides (all three).



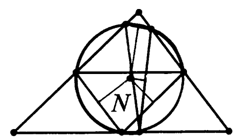
The medial has altitudes
That neatly meet at *O*;



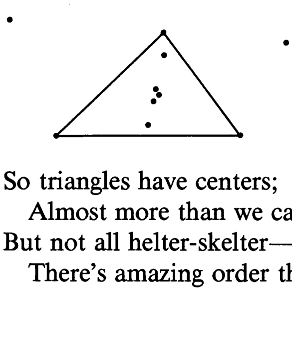
The orthic's angle bisectors
Through *H* are sure to go.



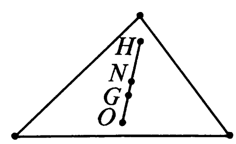
And since the corners of them both
Are nine-point-circle feet,



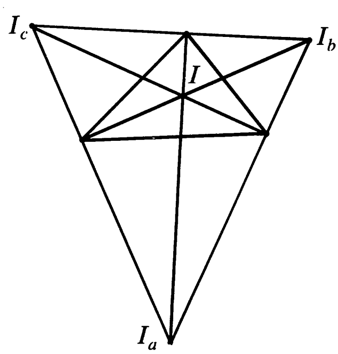
Their perpendicular bisectors
At *N* are sure to meet.



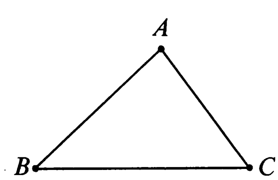
So triangles have centers;
Almost more than we can bear!
But not all helter-skelter—
There's amazing order there.



Four centers on the **Euler line**
Must lie in strict array,
With *N* halfway from *O* to *H*,
And *G* a third away.



Four others on an **orthocentric**
Quadrangle are found:
Incenter in the middle,
With excenters all around.



Who'd ever think a triangle—
Just lowly *ABC*—
Had all these beauties hidden
For us to find and see?