The Kleit Oligopoly Game

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This game is adapted from one developed by Professor Andrew Kleit of Pennsylvania State University.

The Situation

The widget industry in Smallsville has N firms. Each firm produces 150 widgets per month. All costs are fixed, because labor is contracted for on a yearly basis, so we can ignore production cost for the purposes of this case. Widgets are perishable; if they are not sold within the month, they explode in flames.

There are two markets for widgets, the national market, and the local market. The price in the national market is $20 per widget, with the customers paying for delivery, but the price in the local market depends on how many are for sale there in a given month. The price is given by the following market demand curve:

$$P = 100 - \frac{Q}{N},$$

where Q is the total output of widgets sold in the local market. If, however, this equation would yield a negative price, the price is just zero, since the excess widgets can be easily destroyed.

$20 is the **opportunity cost** of selling a widget locally– it is what the firm loses by making that decision. The benefit from the decision depends on what other firms do. All firms make their decisions at the sme time on whether to ship widgets out of town to the national market. The train only comes to Smallsville once a month, so firms cannot retract their decisions. If a firm delays making its decision till too late, then it misses the train, and all its output will have to be sold in Smallsville.

General Procedures

For the first seven months, each of you will be a separate firm. You will write down two things on an index card: (1) the number of the month, and (2) your local-market sales for that month. Also record your local and national market sales on your Scoresheet. The instructor will collect the index cards and then announce the price for that month. You should then calculate your profit for the month and add it to your cumulative total, recording both numbers on your Scoresheet.

For the last five months, you will be organized into five different firms. Each firm has a capacity of 150, and submits a single index card. This card should have the number of the firm on it, as well as the month and the local output.

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If you do not turn in an index card by the deadline, you have missed the train and all 150 of your units must be sold locally.

You can change your decision up until the deadline by handing in a new card noting both your old and your new output, e.g., "I want to change from 40 to 90."

The instructor will calculate the market price, rounding it to the nearest dollar to make computations easier. Note that your own computations will be easier if you pick round numbers for your output.

Procedures Each Month

1. Each student is one firm. No talking.

2. Each student is one firm. No talking.

3. Each student is one firm. No talking.

4. Each student is one firm. No talking.

5. Each student is one firm. No talking.

6. Each student is one firm. You can talk with each other, but then you write down your own output and hand all outputs in separately.

7. Each student is one firm. You can talk with each other, but then you write down your own output and hand all outputs in separately.

8. You are organized into Firms 1 through 5, so N=5. People can talk within the firms, but firms cannot talk to each other. The outputs of the firms are secret.

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10. You are organized into Firms 1 through 5, so N=5. You can talk to anyone you like, but when the talking is done, each firm writes down its output secretly and hands it in. (This month counts for the group bonus)

11. You are organized into Firms 1 through 5, so N=5. You can talk to anyone you like, but when the talking is done, each firm writes down its output secretly and hands it in. Write the number of your firm with your output. This number will be made public once all the outputs have been received. (This month counts for the group bonus)

12. You are organized into Firms 1 through 5, so N=5. People can talk with anyone they like, and arrange to submit outputs jointly if they like. Write the number of your firm with your output. This number will be made public once all the outputs have been received. (This month counts for the group bonus)

Prizes

To add spice to the game, there will be three kinds of bonuses.

First, whichever student has the highest profits over all the months receives 4 extra points on the next quiz.

Second, each member of the team with the highest profits over the team months receives 2 extra points on the next quiz.

Third, if the average profit over all the months exceeds 4,000, each member of the class will receive 3 extra points on the next quiz.

Month	Firm 1	Firm 2	Firm 3	Firm 4	Firm 5	Local Price
1	–	–	–	–	–	
2	–	–	–	–	–	
3	–	–	–	–	–	
4	–	–	–	–	–	
5	–	–	–	–	–	
6	–	–	–	–	–	
7	–	–	–	–	–	
8						
9						
10						
11						
12						

TABLE 1: GAME RESULTS: OUTPUTS AND PRICES

Your Name:
Your Firm:
Total Cumulative Revenue:
(months 1-7 plus 8-12)

Month	1	2	3	4	5	6	7
Your Local Sales							
Local Price							
Local Revenue							
Your National Sales							
National Price	20	20	20	20	20	20	20
National Revenue							
Total Revenue							
Cumulative Revenue							

Month	8	9	10	11	12
Your Local Sales					
Local Price					
Local Revenue					
Your National Sales					
National Price	20	20	20	20	20
National Revenue					
Total Revenue					
Cumulative Revenue					

TABLE 2: SCORESHEET

The local price depends on local output. Total revenue is the month's national revenue plus its local revenue. Cumulative revenue is the sum of that month's revenue plus every preceding month's revenue.

Analysis

If firm i sells Q_i locally, then local sales will be $\Sigma_{i=1}^{N} Q_i$. Industry revenue will be the sum of local and national-market revenues:

$$\Pi = \left(\Sigma_{i=1}^{N} Q_i\right)\left(100 - \frac{\Sigma_{i=1}^{N} Q_i}{N}\right) + \left(150 - \Sigma_{i=1}^{N} Q_i\right)(20).$$

If this is maximized (which can be done using calculus), then the average sales per firm is 40 and the price is 60, for any number N of firms.

(Using calculus: Industry revenue if each firm produces q is $Nq(100 - Nq/N) + N(150 - q)(20)$, which equals $100Nq - Nq^2 + 3000N - 20Nq$. Setting the derivative with respect to q equal to zero yields $100N - 2Nq - 20N = 0$, which can be simplified to $q = 40$).

Suppose, however, that $(N - 1)$ of the firms are selling 40 each, but Firm N is still making up its mind. Firm N faces the following demand curve, which substitutes 40 for the output of each of the other firms:

$$P = 100 - \frac{40(N-1)}{N} - \frac{Q_N}{N} = 100 - 40 + \frac{40}{N} - \frac{Q_N}{N} = 60 - \frac{Q_N - 40}{N}.$$

If $N = 5$, firm N maximizes its profit by choosing $Q_N = 120$, something which also needs calculus to calculate. This means that it may be difficult to create an environment in which each firm sells just 40 locally.

The following table shows what happens at various levels of local sales per firm if all firms maintain the same level.

Local Sales per Firm	**Local Price**	**Local-Market Revenues per Firm**	**National-Market Revenues per Firm**	**Total Revenues per Firm**
0	100	0	3,000	3,000
10	90	900	2,800	3,700
20	80	1,600	2,600	4,200
30	70	2,100	2,400	4,500
40	60	2,000	2,200	4,600
50	50	2,500	2,000	4,500
60	40	2,400	1,800	4,200
70	30	2,100	1,600	3,700
80	20	1,600	1,400	3,000
90	10	900	1,200	2,100
100	0	0	0	0

TABLE 3: PRICES AND OUTPUTS

Cournot Equilibrium

To find the Nash equilibrium (the Cournot equilibrium, this model), set up the payoff function of the individual firm. Suppose all the other firms choose local sales of q, but the first firm chooses q_1. Firm 1's payoff is then

$$\begin{aligned}\pi_1 &= Pq_1 + 20 * (150 - q_1) = (100 - \tfrac{Q}{N})q_1 + 20 * (150 - q_1) \\ &= 100q_1 - \tfrac{((N-1)q+q_1)q_1}{N} + 20 * (150 - q_1) \\ &= 100q_1 - \tfrac{(N-1)qq_1}{N} - \tfrac{q_1^2}{N} + 3000 - 20q_1.\end{aligned}$$

Differentiating with respect to q_1 yields the first order condition

$$\frac{d\pi_1}{dq_1} = 100 - \frac{(N-1)q}{N} - \frac{2q_1}{N} + 0 - 20 = 0,$$

which can be solved to yield $80N = (N-1)q + 2q_1$. If we furthermore guess that the equilibrium is symmetric, so $q = q_1$, then we can write $80N = (N+1)q$, and $q = \frac{80N}{N+1}$.

If $N = 5$, then each firm produces $q = \frac{400}{6} = 66.67$, and the price is 33.33. (I will round all decimals to two places.)

If $N = 20$, then each firm produces $q = \frac{1600}{21} = 76.19$, and the price is 23.8.

If $N = 40$, then each firm produces $q = \frac{3200}{41} = 78.05$, and the price is 21.95.

If $N = 400$, then each firm produces $q = \frac{32000}{401} = 79.80$, and the price is 20.2.

Instructor's Notes

Equipment:
1. A buzzer
2. Index cards
3. A calculator

The first rounds can go by very quickly. They are so the students will learn how the demand curve works. Students will usually start with cautiously low local outputs. Allow more time per month for the later months, since there will be discussion then.

I like to leave an overhead on the projector during the game with what happens in each of the 12 months. While the students are making their decisions, I write a table up on the board to show the outputs and prices. I put up the prices month by month, and then go back at the end of the class and insert the individual firm outputs.

Make the point that in Month 12, cartels are legal, but not inevitable. There is still a holdout problem. If 4 firms agree to produce 40 each, the 5th firm will hold out and produce 80. Or, you might find that total cartel output is 200, but one firm insists that the others each produce 30 and it produce 80.

Also make the point that the Nash equilibrium price is not 20, but 33 1/3. Explain that this is because if a firm deviates and sells more, then it will drive the price down enough that its own profits will fall too. If, however, firms had unlimited capacities *and* they chose prices instead of quantities, the result would be different.

Miscellaneous Notes.

1. Professor Maxwell says that in the Kleit game, he has found prices falling with fewer firms. Variance of outputs falls too. The reason: The risk-takers dominate their new teams.

(Article from WSJ about how women individual investors make more money than men, because they churn less.)